

$$7) y = x^{\frac{1}{3}} (x - 4)$$

$$y' = \frac{1}{3} x^{-\frac{2}{3}} (x - 4) + x^{\frac{1}{3}} (1)$$

$$y' = \frac{x-4}{3x^{\frac{2}{3}}} + x^{\frac{1}{3}} = \frac{x-4+3x}{3x^{\frac{2}{3}}} = \frac{4x-4}{3x^{\frac{2}{3}}} = 0$$

$$4x-4=0 \quad 3x^{\frac{2}{3}}=0$$

$$\boxed{4x=4} \quad | \quad \boxed{x=0}$$

x	-1	0	1	2
y'	-	-	-	+
	↘	↘	↘	↗

تناقص $(-\infty, 0], [0, 1]$

تزايد $[1, \infty)$

صغرى علي $f(1) = -3$

$$y' = \frac{1}{3} x^{-\frac{2}{3}} (4x - 4)$$

$$y'' = -\frac{2}{9} x^{-\frac{5}{3}} (4x - 4) + \frac{1}{3} x^{-\frac{2}{3}} (4)$$

$$y'' = x^{-\frac{5}{3}} \left[-\frac{2}{9} (4x - 4) + \frac{4}{3} x \right]$$

$$y'' = \frac{-\frac{8}{9}x + \frac{8}{9} + \frac{4}{3}x}{x^{\frac{5}{3}}}$$

$$-\frac{8}{9}x + \frac{8}{9} + \frac{4}{3}x = 0$$

$$\frac{4}{9}x + \frac{8}{9} = 0$$

$$\frac{4}{9}x = -\frac{8}{9}$$

$$\boxed{x = -2}$$

$$x^{\frac{5}{3}} = 0$$

$$\boxed{x = 0}$$

x	-3	-2	-1	0	1
y'	+	-	-	-	+
	↘	↘	↘	↘	↗
	$f'' > 0$	$f'' < 0$	$f'' < 0$	$f'' < 0$	$f'' > 0$

تقعر لأس $(-\infty, -2), (0, \infty)$

تقعر لأسفل $(-2, 0)$

انقلاب $x = -2$

انقلاب $x = 0$

8) $y = x^{\frac{3}{4}} (5 - x)$, $x \geq 0$

المجال: $x \geq 0$

$$y' = \frac{3}{4} x^{-1/4} (5 - x) + x^{3/4} (-1)$$

$$y' = x^{-1/4} \left[\frac{3}{4} (5 - x) - x \right]$$

$$y' = x^{-1/4} \left(\frac{15}{4} - \frac{7}{4}x - x \right)$$

$$y' = \frac{15/4 - 7/4x}{x^{1/4}}$$

$$\frac{15}{4} - \frac{7}{4}x = 0, \quad x^{1/4} = 0 \quad \boxed{x=0}$$

$$\frac{15}{4} = \frac{7}{4}x \rightarrow \boxed{x = \frac{15}{7}}$$

x	0	1	3
y'	0	+	-
y			

تزايد $[0, \frac{15}{7}]$

تناقص $[\frac{15}{7}, \infty)$

$f(\frac{15}{7}) = \bigcirc$ عظمى محلية

$$y' = x^{-1/4} (\frac{15}{4} - \frac{7}{4}x) \quad y'' = -\frac{1}{4} x^{-5/4} (\frac{15}{4} - \frac{7}{4}x) + x^{-1/4} (-\frac{7}{4})$$

$$y'' = x^{-5/4} \left(-\frac{15}{16} + \frac{7}{16}x - \frac{7}{4}x \right) = x^{-5/4} \left(\frac{21}{16}x - \frac{15}{16} \right)$$

$$y'' = \frac{21/16 x - 15/16}{x^{5/4}} = 0$$

$$\frac{21}{16}x = \frac{15}{16}$$

$$x = \frac{15}{21}$$

$$\boxed{x = \frac{5}{7}}$$

$$x^{5/4} = 0$$

$$\boxed{x=0}$$

x	0	1/2	5/7	1
y''	0	-	+	+
y				

تقعر لأسفل $(0, \frac{5}{7})$

تقعر لأعلى $(\frac{5}{7}, \infty)$

نقطة انعطاف $x = \frac{5}{7}$