اجابات الأسبوع الخامس

السؤال الأول :

إذا كانت $$I = \int_0^6 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{6-x}}\,dx$$

أولا : باستخدام التعويض بيّن أن

$$I = \int_0^6 \frac{\sqrt{6-x}}{\sqrt{x}+\sqrt{6-x}}\,dx$$

الحل : $6 - x = u \to\to \; dx = -du$

$$I = \int_0^6 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{6-x}}\,dx$$

$$I = \int_6^0 \frac{-\sqrt{6-u}}{\sqrt{6-u}+\sqrt{u}}\,du = \int_0^6 \frac{\sqrt{6-x}}{\sqrt{x}+\sqrt{6-x}}\,dx$$

ثانيا : :باستخدام ما حصلت عليه في أولا أوجد I

الحل :

$$I + I = \int_0^6 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{6-x}}\,dx + \int_0^6 \frac{\sqrt{6-x}}{\sqrt{x}+\sqrt{6-x}}\,dx$$

$$2 \times I = \int_0^6 \frac{\sqrt{x}+\sqrt{6-x}}{\sqrt{x}+\sqrt{6-x}}\,dx = \int_0^6 1\,dx = 6$$

$$I = 3$$

ثالثا: أوجد بنفس الطريقة

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}\,dx$$

$$x = \frac{\pi}{2} - u \qquad \sin x = sin(\frac{\pi}{2} - u) \quad \cos x = \cos\left(\frac{\pi}{2} - u\right)$$

$$x = 0 \to\to u = \frac{\pi}{2} \quad , x = \frac{\pi}{2} \ \to u = 0 \to dx = -du$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = -\int_{\frac{\pi}{2}}^{0} \frac{\sin\left(\frac{\pi}{2} - u\right)}{\sin\left(\frac{\pi}{2} - u\right) + \cos\left(\frac{\pi}{2} - u\right)} du$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = \ -\int_{\frac{\pi}{2}}^{0} \frac{cosu}{cosu + sinu} du$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = \int_0^{\frac{\pi}{2}} \frac{cosu}{cosu + sinu} du = \int_0^{\frac{\pi}{2}} \frac{cosx}{cosx + sinx} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{cosx}{cosx + sinx} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x + cosx}{\sin x + \cos x} dx = \int_0^{\frac{\pi}{2}} 1\, dx = \frac{\pi}{2} \qquad \to\to\to \quad I = \frac{\pi}{4}$$

السؤال الثاني أولا : أثبت أن $\int e^x\left(f(x) + f^{/}(x)\right) dx = \ e^x f(x) + c$

الحل : نشتق الطرف الثاني لنبرهن أنه يساوي المشتقة العكسية

للدالة $e^x\left(f(x) + f^{/}(x)\right)$

$$(e^x f(x))^{/} = e^x f(x) + e^x f^{/}(x)$$

$$= e^x\left(f(x) + f^{/}(x)\right)$$

ثانيا : أستخدم النتيجة السابقة في إيجاد $\int \frac{xe^x}{(x+1)^2} dx$

$$\int \frac{xe^x}{(x+1)^2} dx = \int \frac{(x+1-1)e^x}{(x+1)^2} dx$$

$$= \int e^x \left(\frac{(x+1)}{(x+1)^2} - \frac{1}{(x+1)^2}\right) dx$$

$$= \int e^x \left(\frac{1}{x+1} + \frac{-1}{(x+1)^2}\right) dx$$

بفرض أن $f(x) = \frac{1}{x+1}$ يكون

$$\int e^x \left(\frac{1}{x+1} + \frac{-1}{(x+1)^2}\right) dx = \int e^x \left(f(x) + f^{/}(x)\right) dx = e^x f(x) + c$$

ثالثا : أوجد بطريقة الاستخدام المتكرر $\int sin(lnx)\, dx$

هل يجوز استخدام التكامل الجدولي في هذا التكامل ؟ برر إجابتك ؟

الحل : $dx = dv \to\to\to x = v \quad , \quad u = sin(lnx) \to\to du = \frac{cos(lnx)}{x}$

$$\int sin(lnx)\, dx = xsin(lnx) - \int \frac{xcos(lnx)}{x} dx$$

$$m = cos(lnx) \to\to dm = \frac{-sin(lnx)}{x}, \quad dx = dn \to\to\to x = n$$

$$\int sin(lnx)\, dx = xsin(lnx) - \left(xcos(lnx) + \int \frac{xsin(lnx)}{x} dx\right)$$

$$2\int sin(lnx)\, dx = xsin(lnx) - xcos(lnx)$$

$$\int sin(lnx)\, dx = \frac{1}{2}\left(xsin(lnx) - xcos(lnx)\right)$$

السؤال الثالث

إذا كان $I = \int_0^{\frac{\pi}{4}} (cosx)^4\, dx$ **و** $J = \int_0^{\frac{\pi}{4}} (sinx)^4\, dx$

أولا : العلاقة $(sinx)^4 + (cosx)^4 = \frac{3}{4} + \frac{\cos 4x}{4}$

الحل : $(cos2x)^2 = ((cosx)^2 - (sinx)^2)^2$

$$= (cosx)^4 + (sinx)^4 - 2(cosx)^2(sinx)^2$$

$$(cosx)^4 + (sinx)^4 = (cos2x)^2 + 2(cosx)^2(sinx)^2$$

$$(cosx)^4 + (sinx)^4 = (cos2x)^2 + 2\left(\frac{1+cos2x}{2}\right)\left(\frac{1-cos2x}{2}\right)$$

$$(cosx)^4 + (sinx)^4 = (cos2x)^2 + \frac{1}{2}(1-(cos2x)^2)$$

$$= \frac{1}{2} + \frac{1}{2}(cos2x)^2$$

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{1+cos4x}{2}\right) = \frac{3}{4} + \frac{cos4x}{4}$$

ثانيا : أوجد قيمة $J + I$ و $I - J$

$$I + J = \int_0^{\frac{\pi}{4}} ((cosx)^4 + (sinx)^4)\, dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{3}{4} dx + \frac{1}{4}\int_0^{\frac{\pi}{4}} cos4x\, dx = \frac{3}{16}\pi + \frac{1}{16}[sin4x]_0^{\frac{\pi}{4}} = \frac{3}{16}\pi$$

$$I - J = \int_0^{\frac{\pi}{4}} ((cosx)^4 - (sinx)^4)\, dx$$

$$= \int_0^{\frac{\pi}{4}} ((cosx)^2 - (sinx)^2)((cosx)^2 + (sinx)^2)\, dx$$

$$I - J = \int_0^{\frac{\pi}{4}} ((cosx)^2 - (sinx)^2)\, dx = \int_0^{\frac{\pi}{4}} (cos2x)\, dx$$

$$= \frac{1}{2}[sin2x]_0^{\frac{\pi}{4}} = \frac{1}{2}$$

ثالثا : أوجد قيمة كل من J و I

الطريقة الأولى $I - J = \frac{1}{2}$ و $I + J = \frac{3}{16}\pi$ بجمع العلاقتين معا نجد

$$2I = \frac{3}{16}\pi + \frac{1}{2} \quad \to\to \quad I = \frac{3}{32}\pi + \frac{1}{4} = \frac{3\pi+8}{32}$$

$$J = \frac{3}{16}\pi - \left(\frac{3}{32}\pi + \frac{1}{4}\right) = \frac{3\pi-8}{32}$$

الطريقة الثانية :

$$J=\int_0^{\frac{\pi}{4}}(sinx)^4\,dx=\int_0^{\frac{\pi}{4}}((sinx)^2)^2\,dx=\frac{1}{4}\int_0^{\frac{\pi}{4}}(1-cos2x)^2\,dx$$

$$J=\frac{1}{4}\int_0^{\frac{\pi}{4}}(1-2cos2x+(cos2x)^2)\,dx$$

$$\frac{1}{4}\left(\frac{\pi}{4}-[sin2x]_0^{\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}}\frac{1+cos4x}{2}\,dx\right)=\left(\frac{\pi}{16}-\frac{1}{4}\right)+\frac{1}{4}\int_0^{\frac{\pi}{4}}\frac{1+cos4x}{2}\,dx$$

$$=\left(\frac{\pi}{16}-\frac{1}{4}\right)+\frac{1}{8}\int_0^{\frac{\pi}{4}}(1+cos4x)\,dx=\left(\frac{\pi}{16}-\frac{1}{4}\right)+\frac{1}{8}\left[\frac{\pi}{4}+\frac{1}{4}[sin4x]_0^{\frac{\pi}{4}}\right]$$

$$\left(\frac{\pi}{16}-\frac{1}{4}\right)+\frac{1}{8}\left(\frac{\pi}{4}+0\right)=\frac{3\pi}{32}-\frac{1}{4}=\frac{3\pi-8}{32}$$

$$I=\int_0^{\frac{\pi}{4}}(cosx)^4\,dx=\int_0^{\frac{\pi}{4}}((cos)^2)^2\,dx=\frac{1}{4}\int_0^{\frac{\pi}{4}}(1+cos2x)^2\,dx$$

$$J=\frac{1}{4}\int_0^{\frac{\pi}{4}}(1+2cos2x+(cos2x)^2)\,dx$$

$$\frac{1}{4}\left(\frac{\pi}{4}+[sin2x]_0^{\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}}\frac{1+cos4x}{2}\,dx\right)=\left(\frac{\pi}{16}+\frac{1}{4}\right)+\frac{1}{4}\int_0^{\frac{\pi}{4}}\frac{1+cos4x}{2}\,dx$$

$$=\left(\frac{\pi}{16}+\frac{1}{4}\right)+\frac{1}{8}\int_0^{\frac{\pi}{4}}(1+cos4x)\,dx=\left(\frac{\pi}{16}+\frac{1}{4}\right)+\frac{1}{8}\left[\frac{\pi}{4}+\frac{1}{4}[sin4x]_0^{\frac{\pi}{4}}\right]$$

$$\left(\frac{\pi}{16}+\frac{1}{4}\right)+\frac{1}{8}\left(\frac{\pi}{4}+0\right)=\frac{3\pi}{32}+\frac{1}{4}=\frac{3\pi+8}{32}$$