

9.3 Find and Interpret the Slope and the y-intercept from a Straight Line Graph

A Finding the Slope and the y-intercept from a Graph

Remember

The *slope-intercept* form of the equation of a line is $y = mx + b$.

m is the slope and b is the y-intercept.

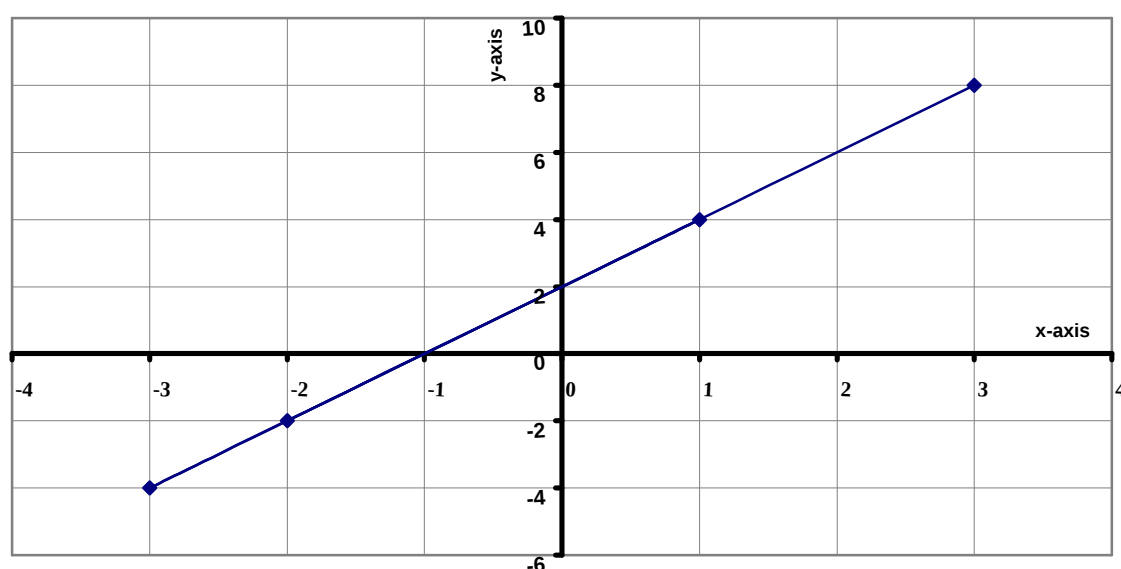
If we know the slope of a line, and the y-intercept, we can write down the equation of the line. Both the slope and the y-intercept can be found from the graph of the line.

To find the slope choose any two points on the line (x_1, x_2) and (y_1, y_2) and substitute the values into the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

To find the y-intercept find the point where the line crosses the y-axis.

Example 1

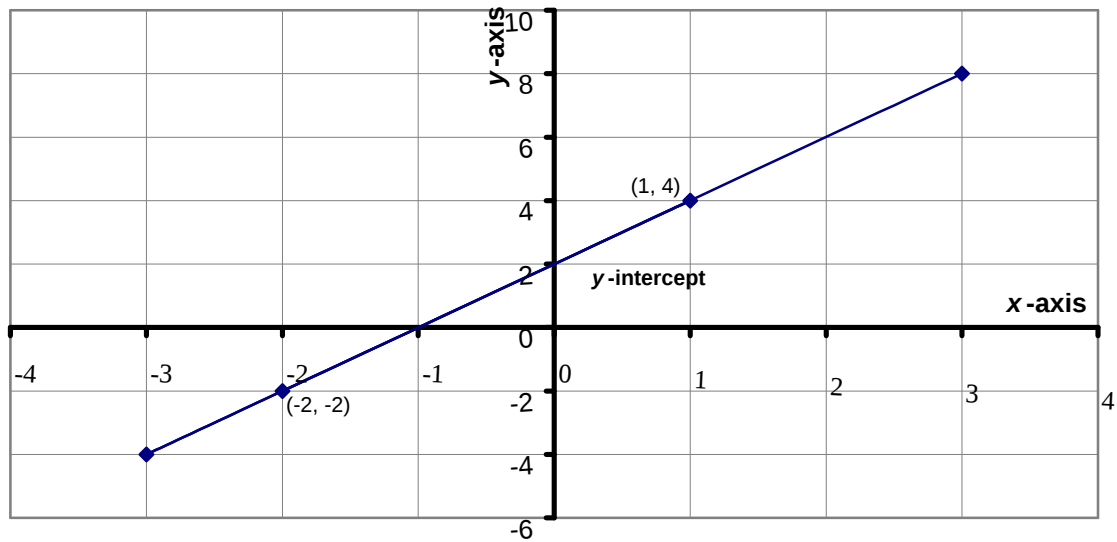
Find the slope and the y-intercept of the line shown below. Write the equation of the line.



Step 1 Choose any two points on the line whose coordinates are easy to read, say $(1, 4)$ and $(-2, -2)$.

Step 2 The slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{-2 - 1} = \frac{-6}{-3} = 2$

Step 3 The line cuts the y -axis at the point $(0, 2)$.
The y -intercept is 2.



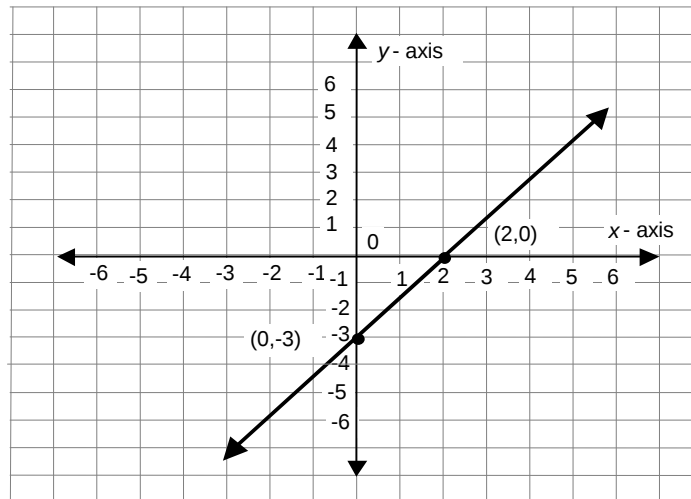
Step 4 Substitute the values of the slope (m) and the y -intercept (b) into the slope-intercept form of the equation of a line $y = mx + b$.

$m = 2$ and $b = 2$. Therefore, the equation of the line is $y = 2x + 2$

Example 2

Find the slope and the y-intercept of the line shown below. Write the equation of the line.

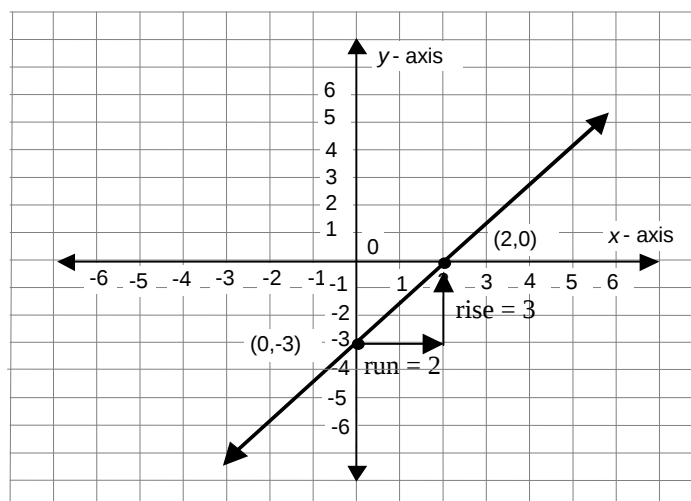
Let's use the run-rise method for this example.



Step1 Choose any two points on the line, say $(0, -3)$ and $(2, 0)$.

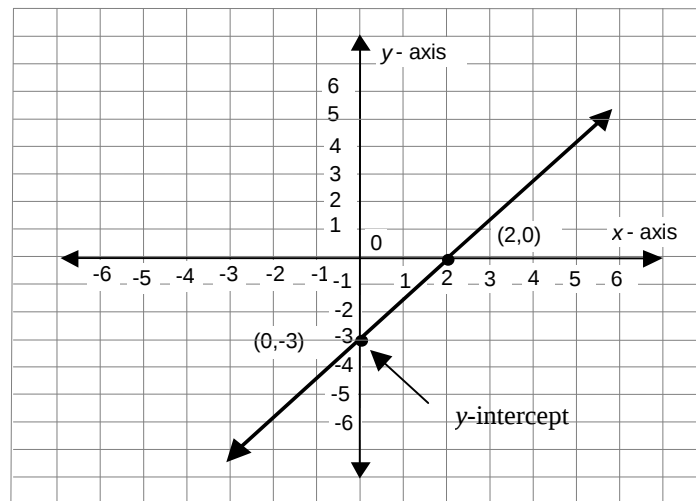
Step 2 To go from $(0, -3)$ to $(2, 0)$, move 2 units to the right run = 2
and go up 3 units rise = 3

$$\text{The slope } m = \frac{\text{rise}}{\text{run}} = \frac{3}{2}$$



Step 3

The line cuts the y -axis at the point $(0, -3)$. The y -intercept is -3 .



Step 4

Substitute the values of the slope (m) and the y -intercept (b) into the slope-intercept form of the equation of a line $y = mx + b$.

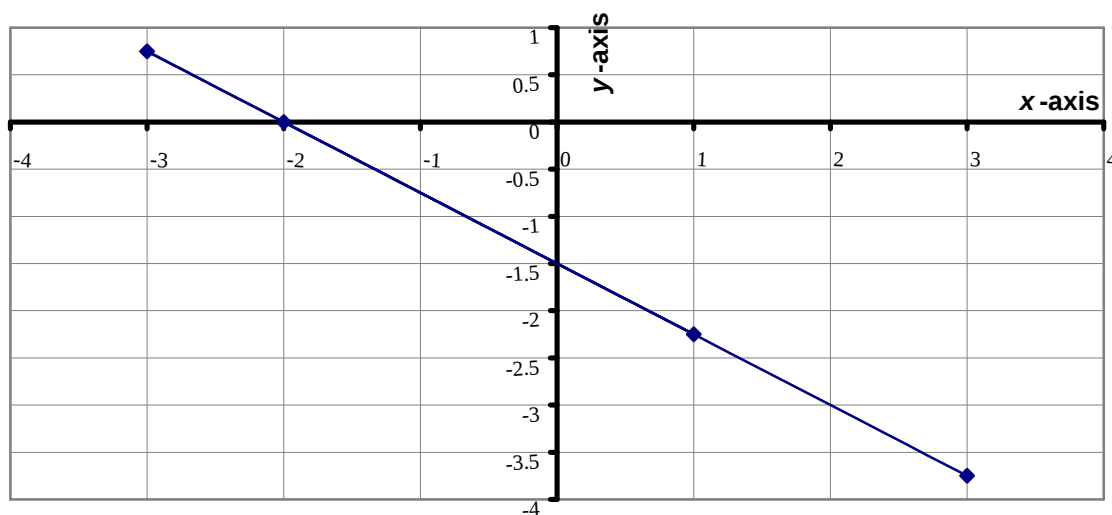
$$m = \frac{3}{2} \text{ and } b = -3.$$

Therefore, the equation of the line is $y = \frac{3}{2}x - 3$

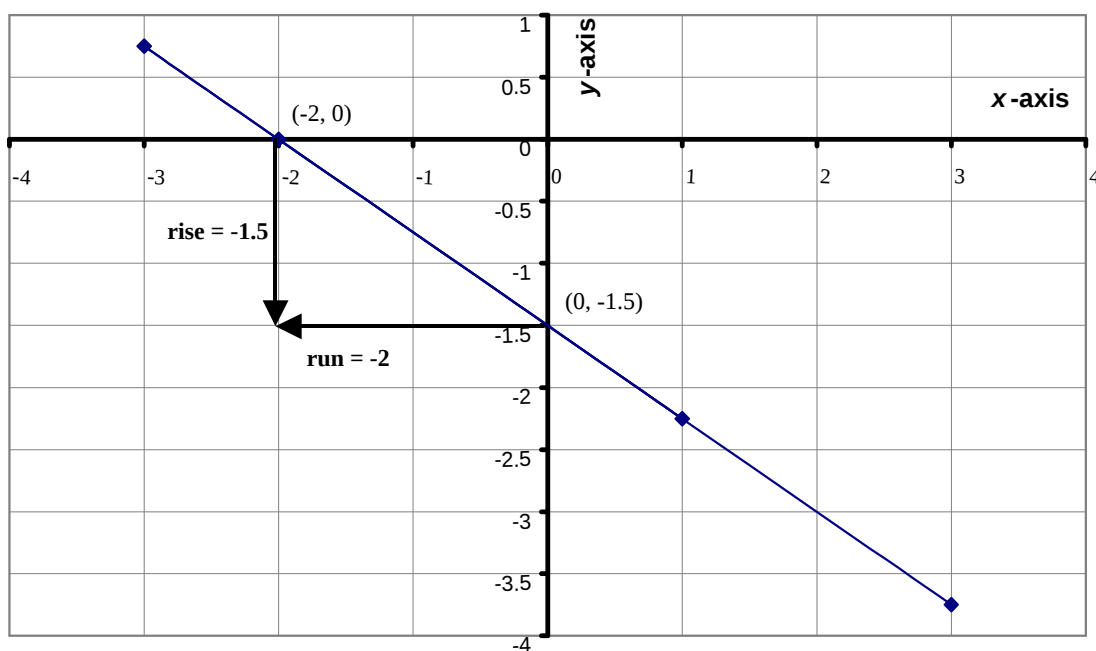
Note:	run > 0	when we move from left to right.
	run < 0	when we move from right to left.
	rise > 0	when we go up.
	rise < 0	when we go down.

Example 3

Find the slope and the y-intercept of the line shown below. Write the equation of the line.

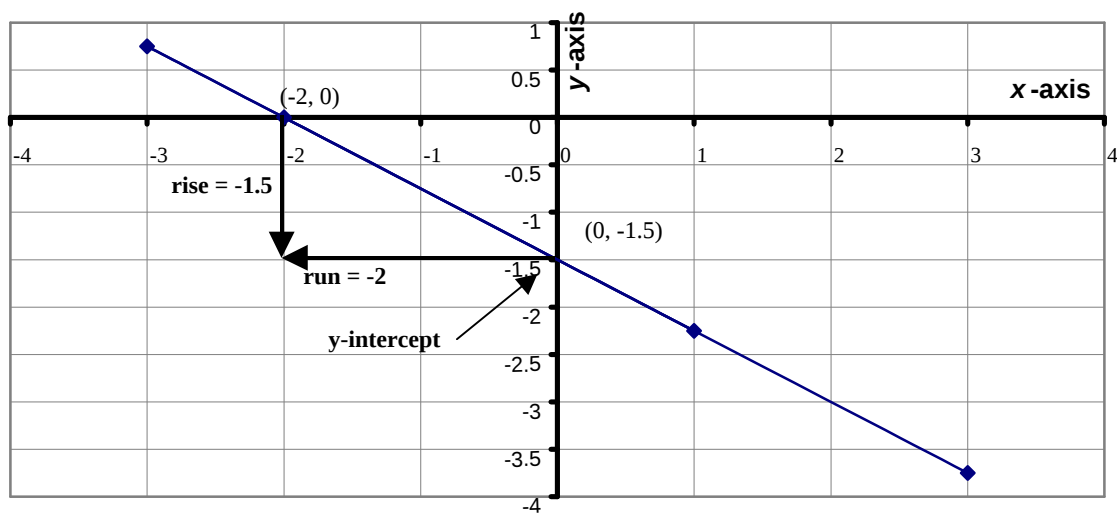


Step 1 Choose any two points on the line, say $(0, -1.5)$ and $(-2, 0)$.



Step 2 The slope $m = \frac{-1.5 - 0}{0 - (-2)} = -\frac{1.5}{2} = -0.75$

Step 3 The line cuts the y-axis at the point $(0, -1.5)$. The y-intercept is -1.5 .



Step 4

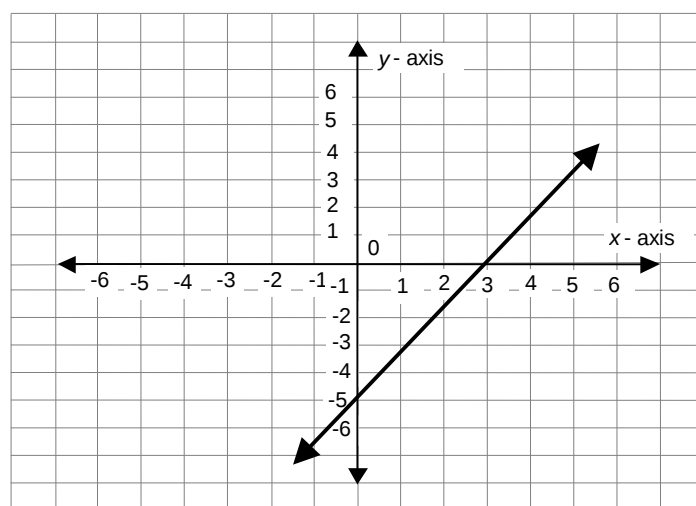
$m = -0.75$ and $b = -1.5$.

Therefore, the equation of the line is $y = -0.75x - 1.5$

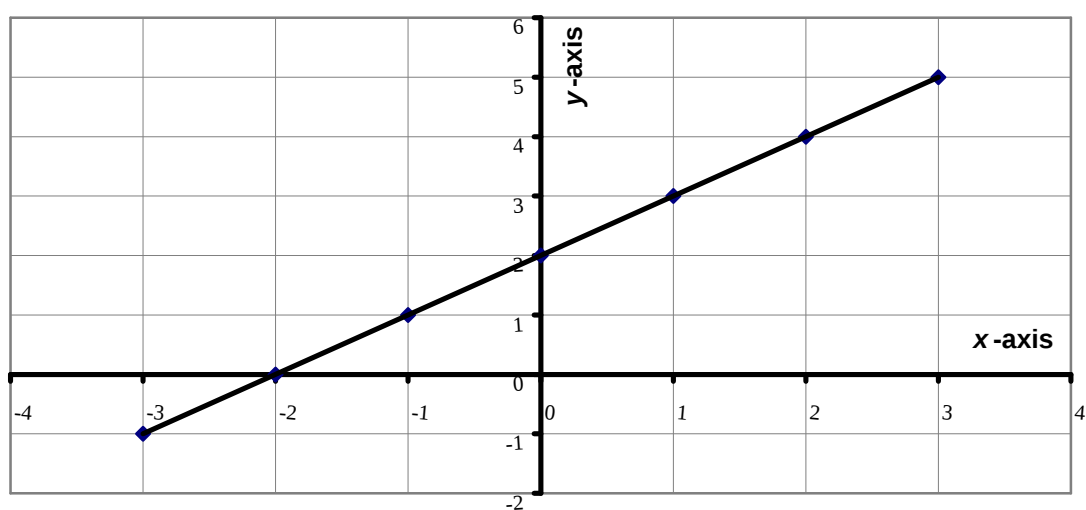
Try these 1

Find the slopes and the y-intercepts of the lines shown below. Write the equations of the lines.

(a)



(b)



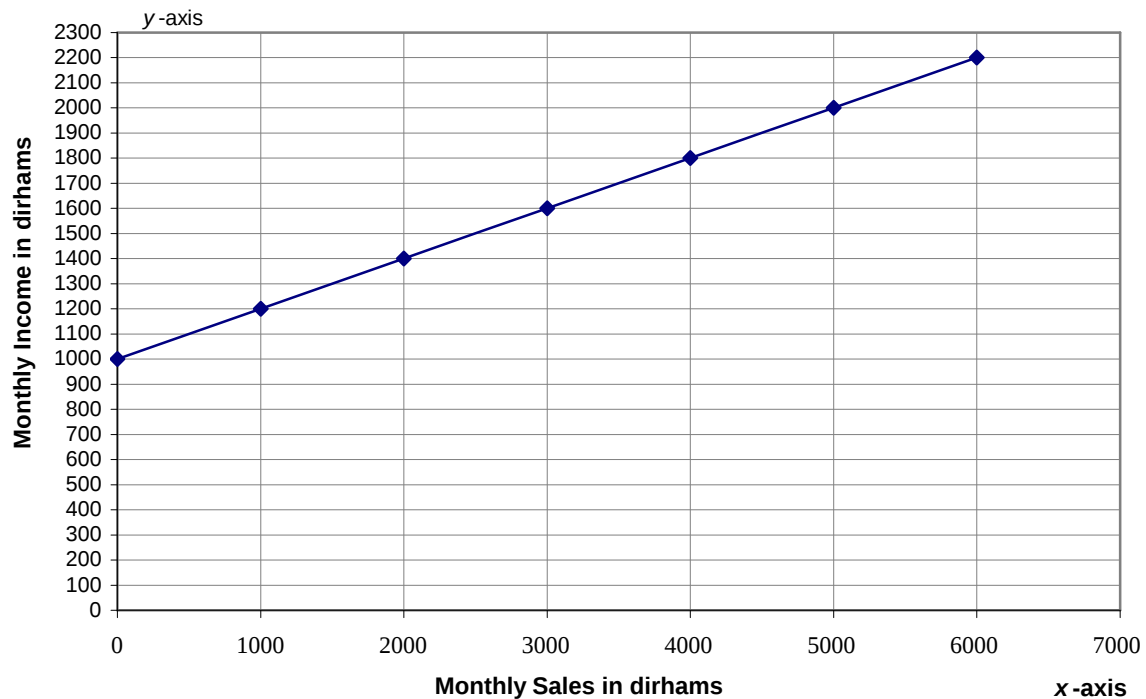
B Applications

Example 4

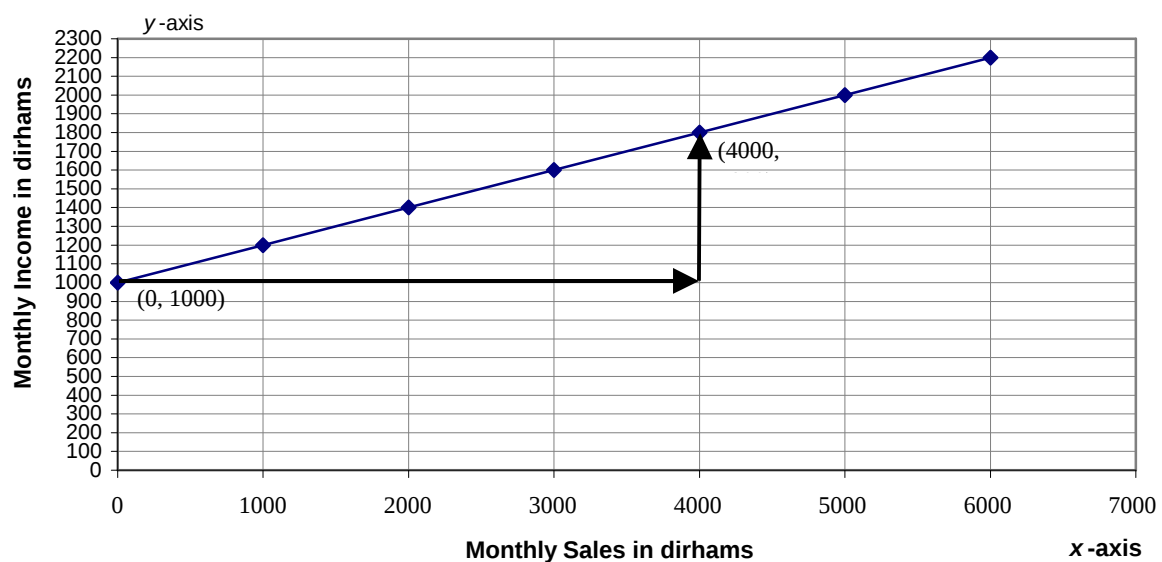
Find the slope and the y -intercept of the line shown below. Write the equation of the line.

In this graph x is the monthly sales in dirhams and y is the monthly income in dirhams.

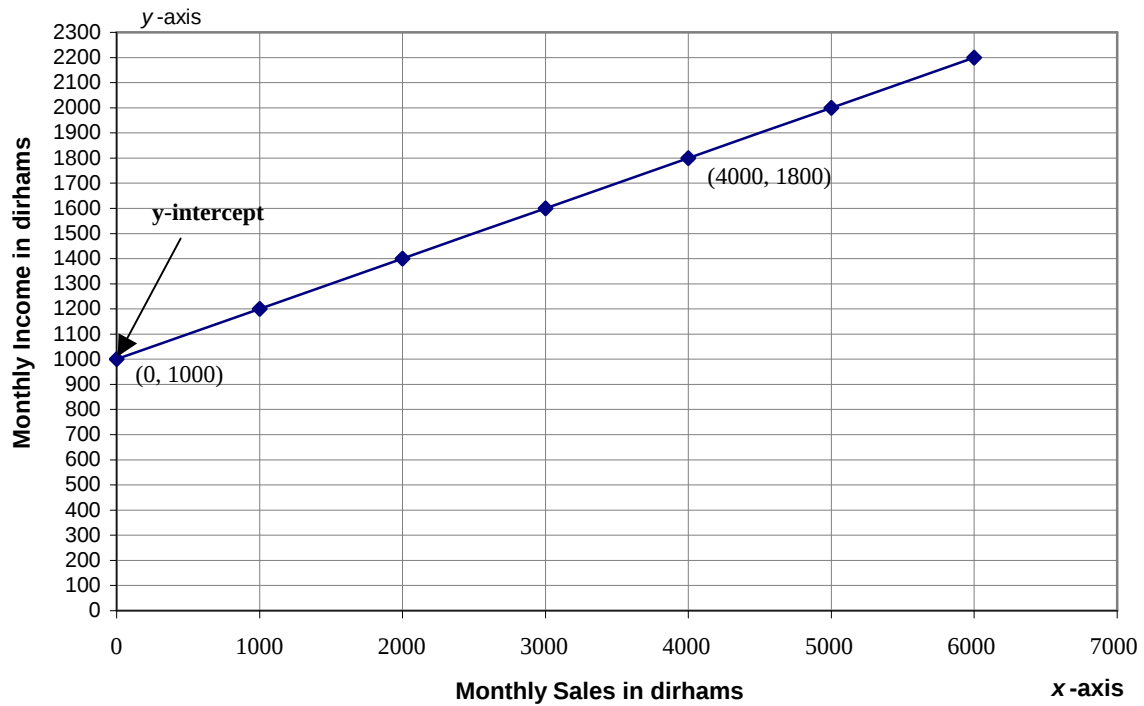
Sales and income can never be negative, thus, we consider only positive values for x and y .



Step 1 Choose any two points on the line, say $(0, 1000)$ and $(4000, 1800)$.



Step 2 The slope $m = \frac{1800 - 1000}{4000 - 0} = \frac{800}{4000} = 0.2$



Step 3 The line cuts the y-axis at the point (0, 1000). The y-intercept is 1000.

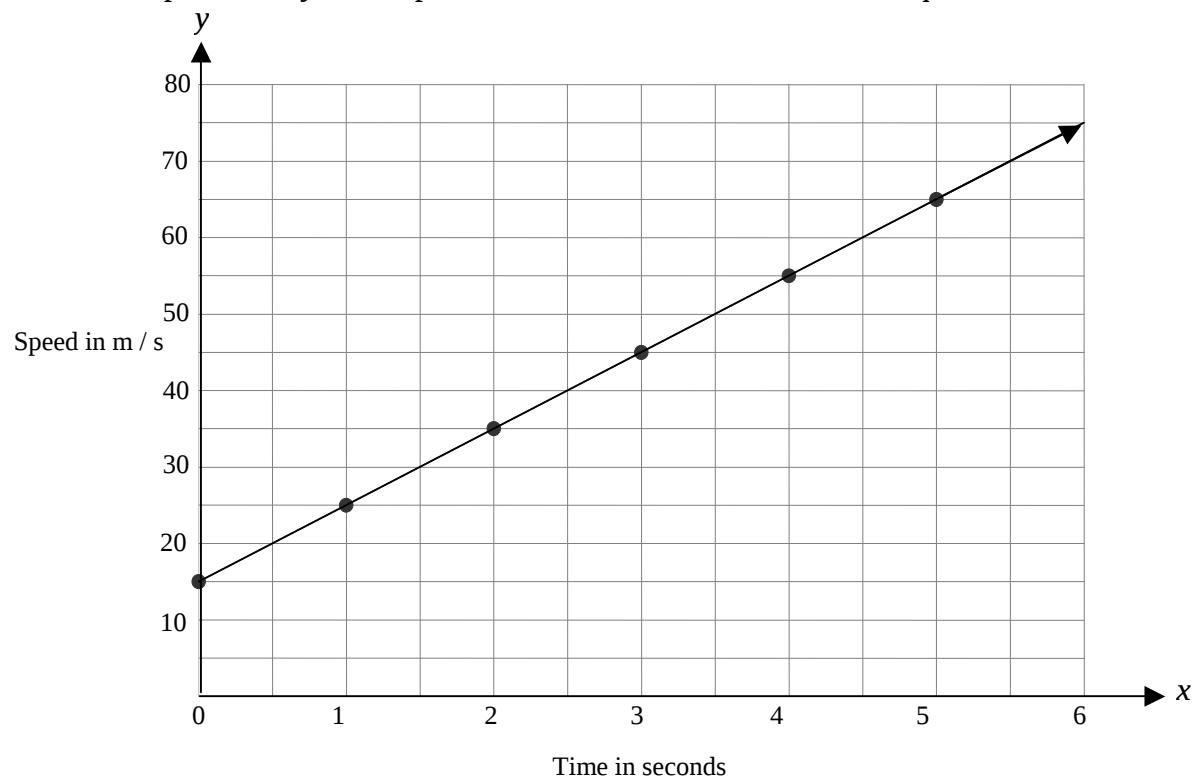
Step 4 Substitute the values of the slope (m) and the y-intercept (b) into the slope-intercept form of the equation of a line $y = mx + b$.

$m = 0.2$ and $b = 1000$.

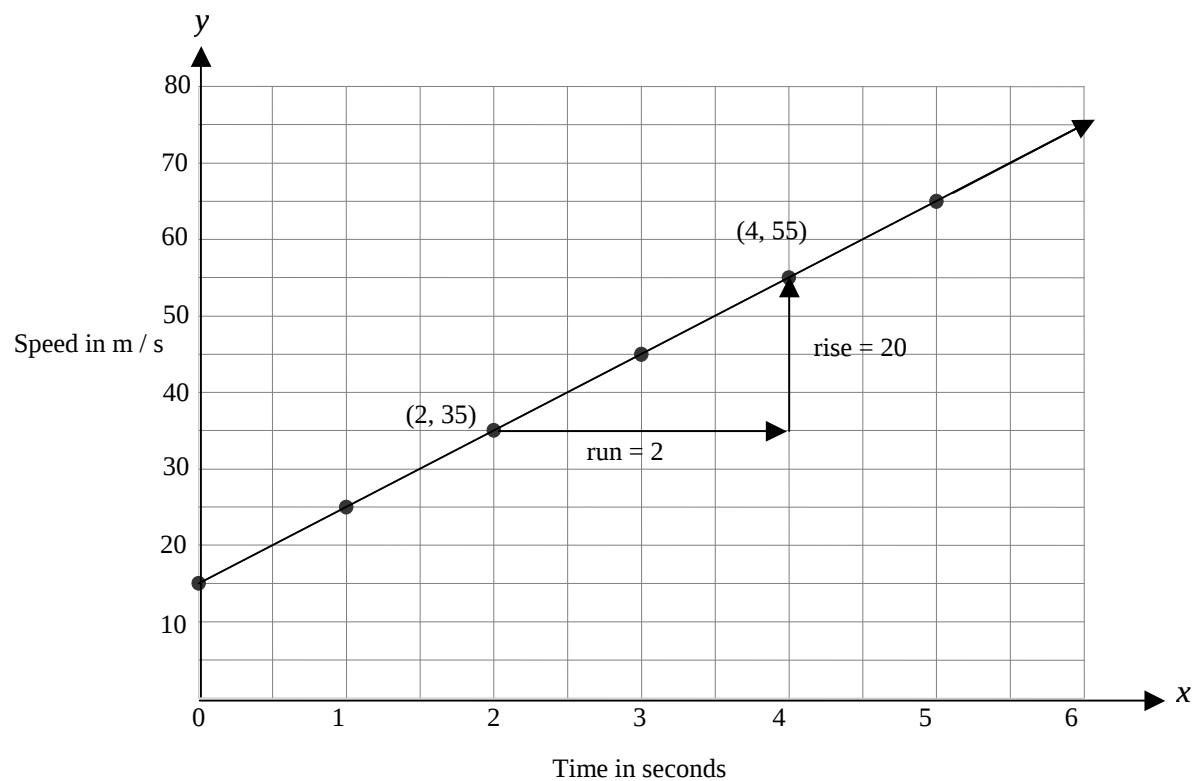
Therefore, the equation of the line is $y = 0.2x + 1000$

Example 5

Find the slope and the y-intercept of the line shown below. Write the equation of the line.

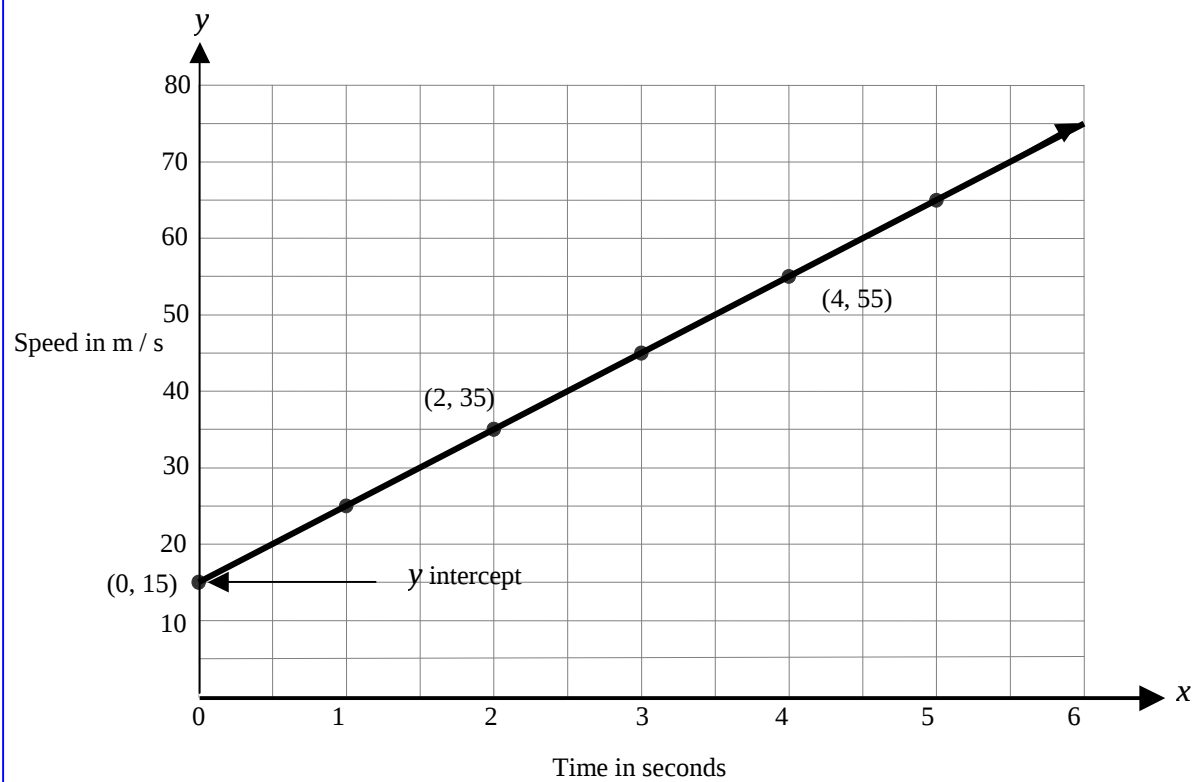


Step 1 Choose any two points on the line, say (2, 35) and (4, 55).



Step 2 $y_2 - y_1 = 55 - 35 = 20$ and $x_2 - x_1 = 4 - 2 = 2$

The slope $m = \frac{20}{2} = 10$



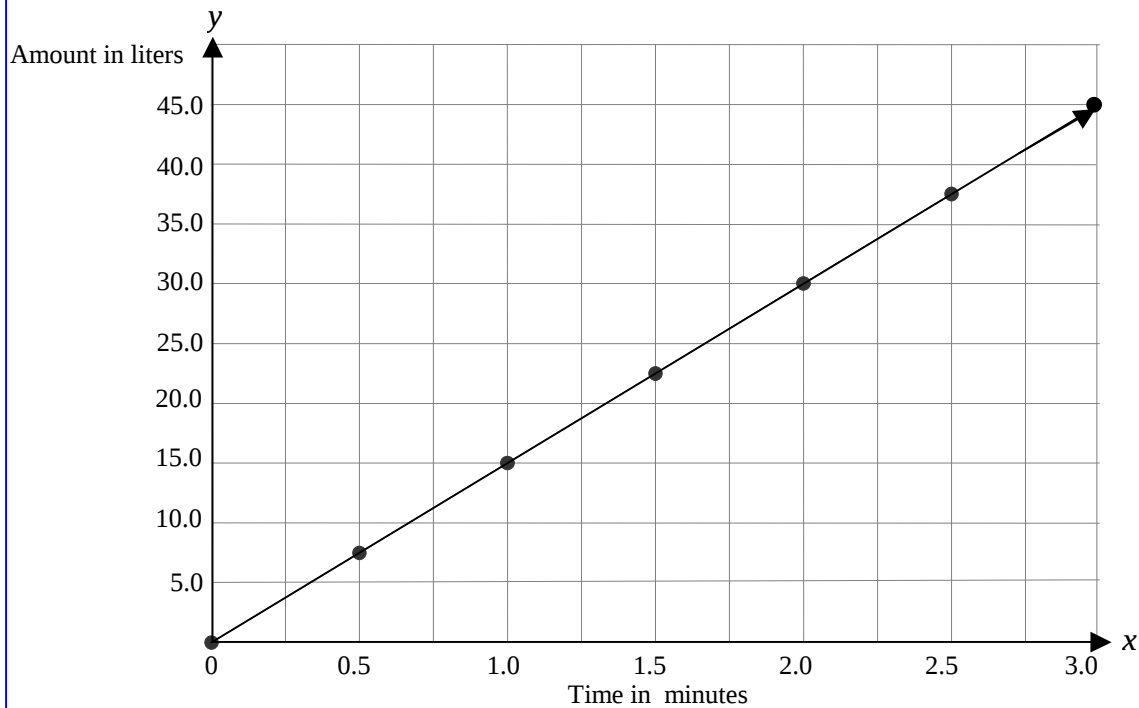
Step 3 The line cuts the y -axis at the point (0, 15). The y -intercept is 15.

Step 4 $m = 10$ and $b = 15$.

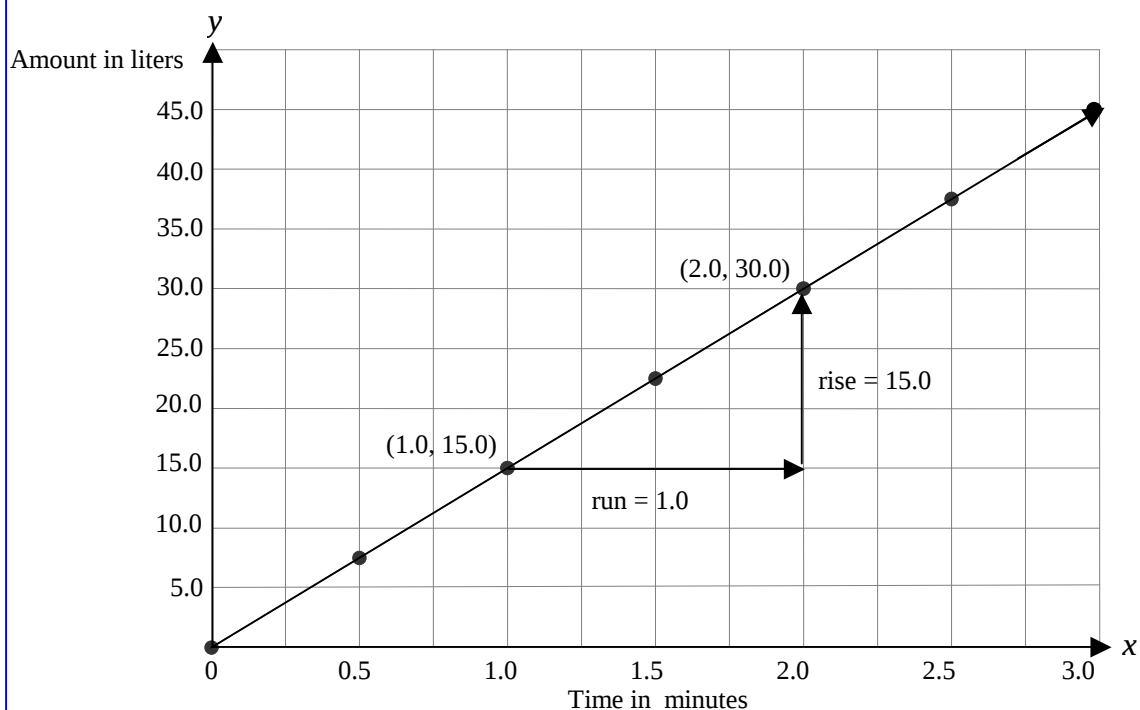
Therefore, the equation of the line is $y = 10x + 15$

Example 6

Water comes out of a bath tap at the rate of 15 liters per minute. The following graph shows the amount of water in the bath at different times. Find the slope and the y-intercept. Write the equation of the line.

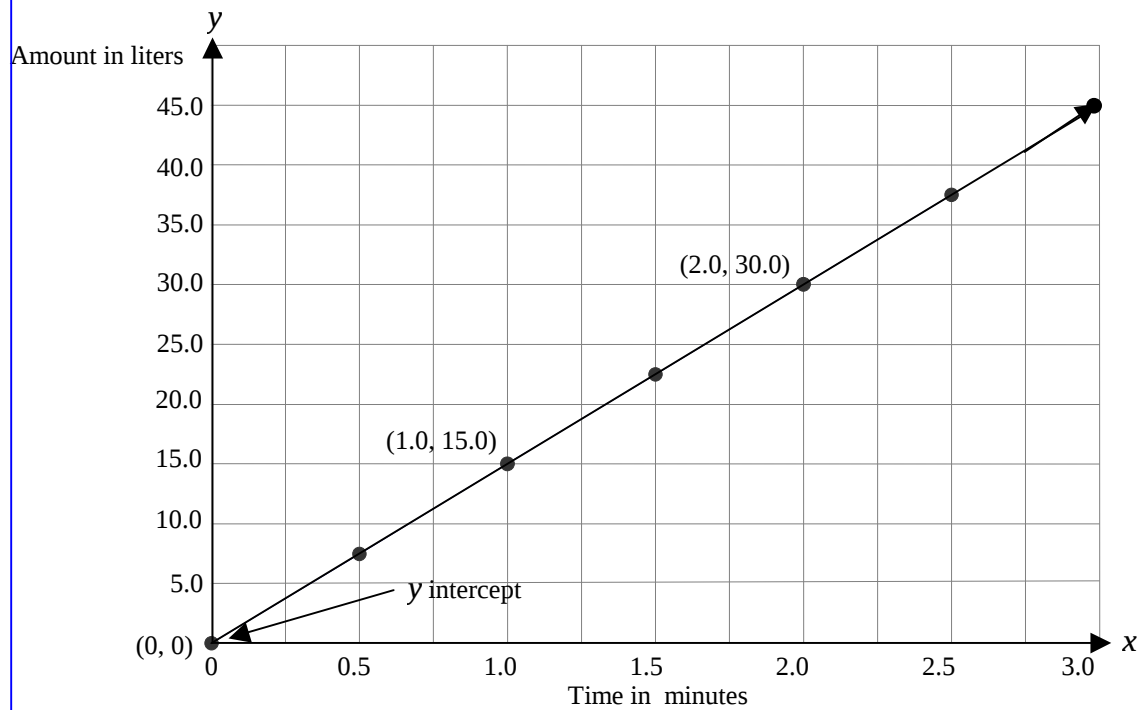


Step 1 Choose any two points on the line, say (1.0, 15.0) and (2.0, 30.0).



Step 2 $y_2 - y_1 = 30.0 - 15.0 = 15.0$ and $x_2 - x_1 = 2.0 - 1.0 = 1.0$

The slope $m = \frac{15.0}{1.0} = 15.0$

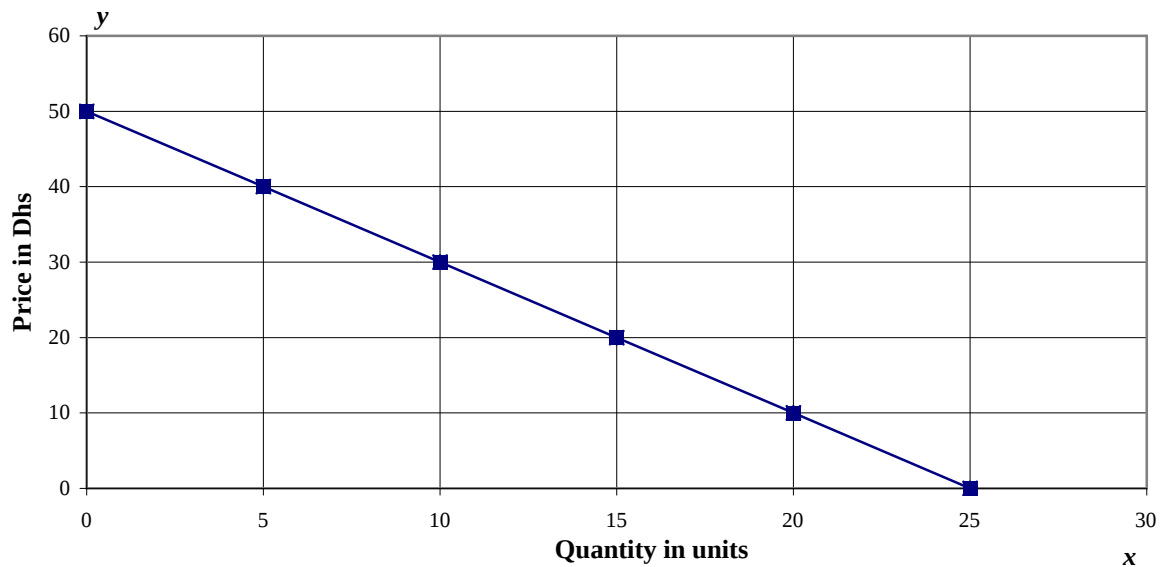


Step 3 The line cuts the y-axis at the point (0, 0). The y-intercept is 0.

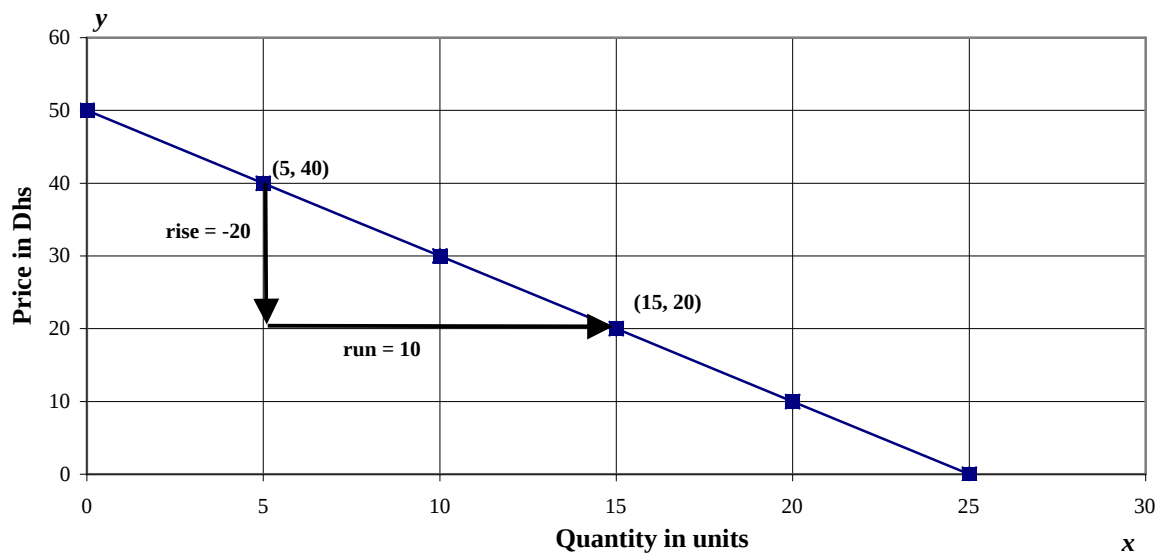
Step 4 $m = 15$ and $b = 0$.
Therefore, the equation of the line is $y = 15x$

Example 7

The following graph shows the price of a certain product at different levels of demand. Find the slope and the y-intercept. Write the equation of the line.

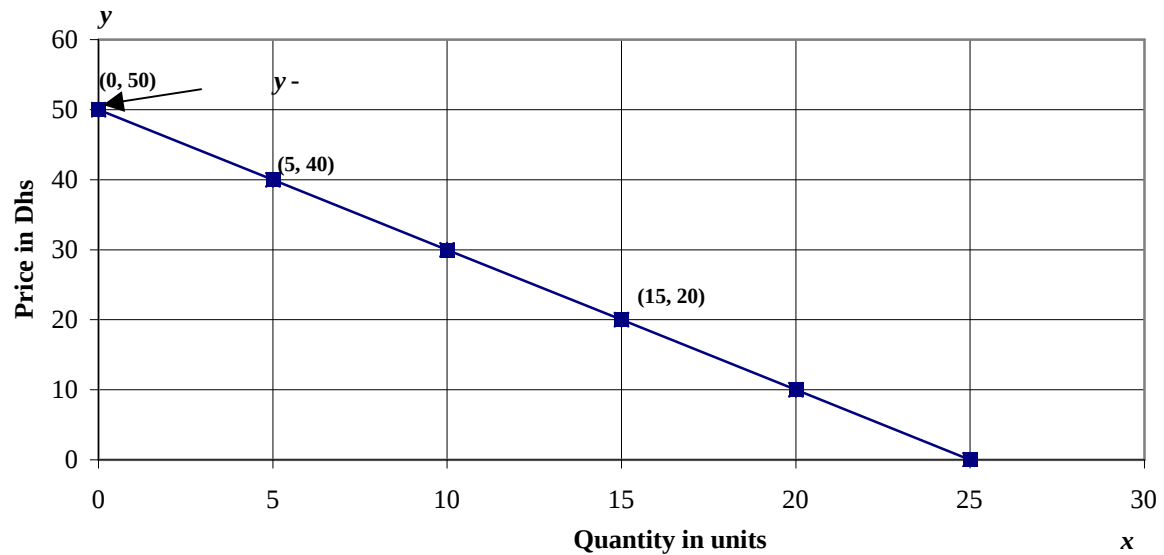


Step 1 Choose any two points on the line, say (5, 40) and (15, 20).



Step 2 $y_2 - y_1 = 20 - 40 = -20$ and $x_2 - x_1 = 15 - 5 = 10$

The slope $m = \frac{-20}{10} = -2$

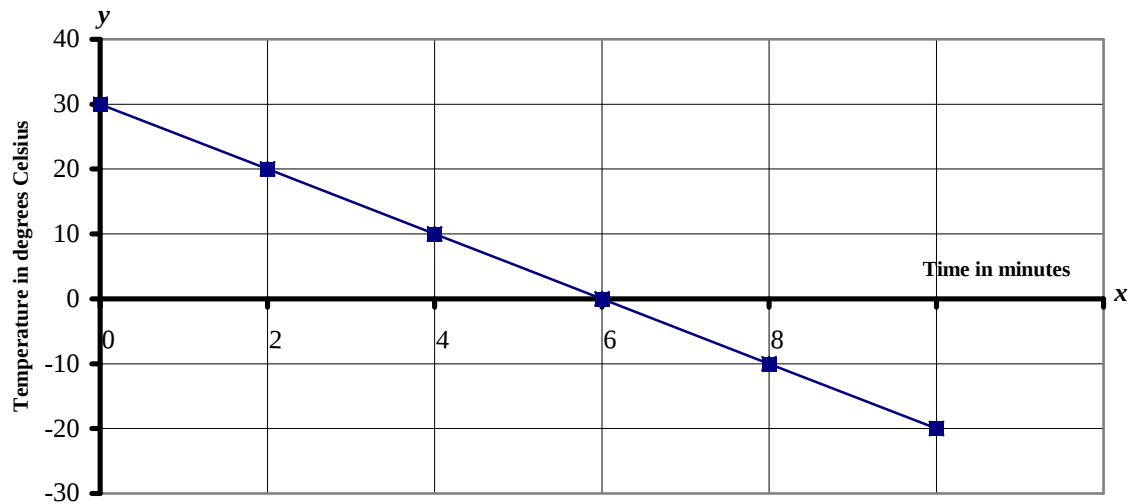


Step 3 The line cuts the y-axis at the point (0, 50). The y-intercept is 50.

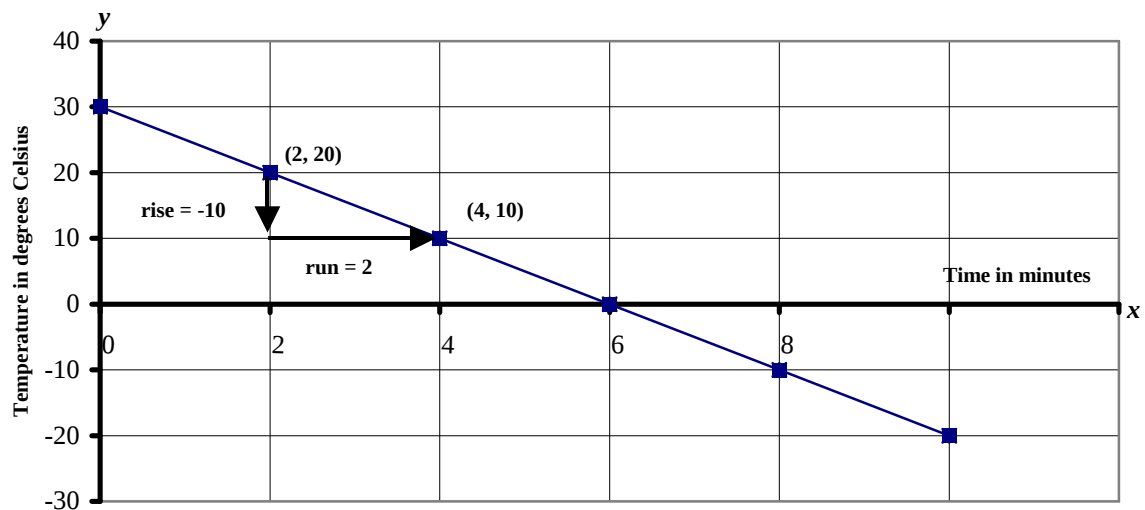
Step 4 $m = -2$ and $b = 50$.
Therefore, the equation of the line is $y = -2x + 50$

Example 8

The following graph shows the temperature of an object at different times. Find the slope and the y-intercept. Write the equation of the line.

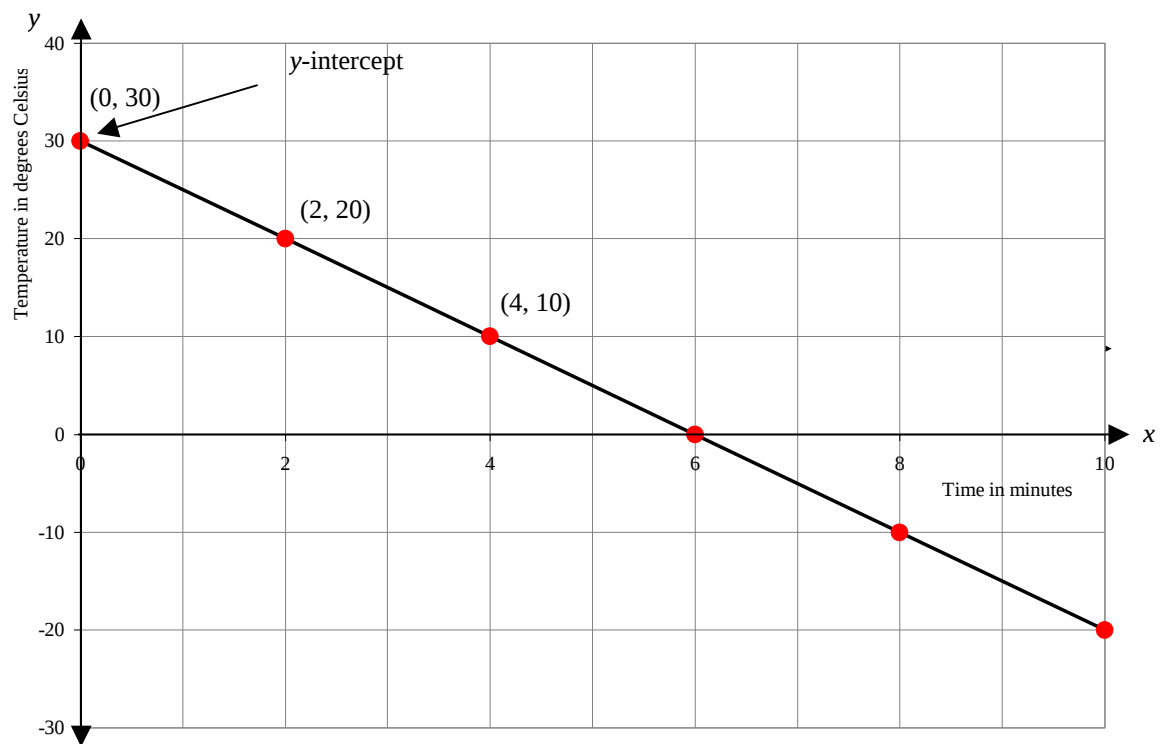


Step 1 Choose any two points on the line, say (2, 20) and (4, 10).



Step 2 $y_2 - y_1 = 10 - 20 = -10$ and $x_2 - x_1 = 4 - 2 = 2$

The slope $m = \frac{-10}{2} = -5$

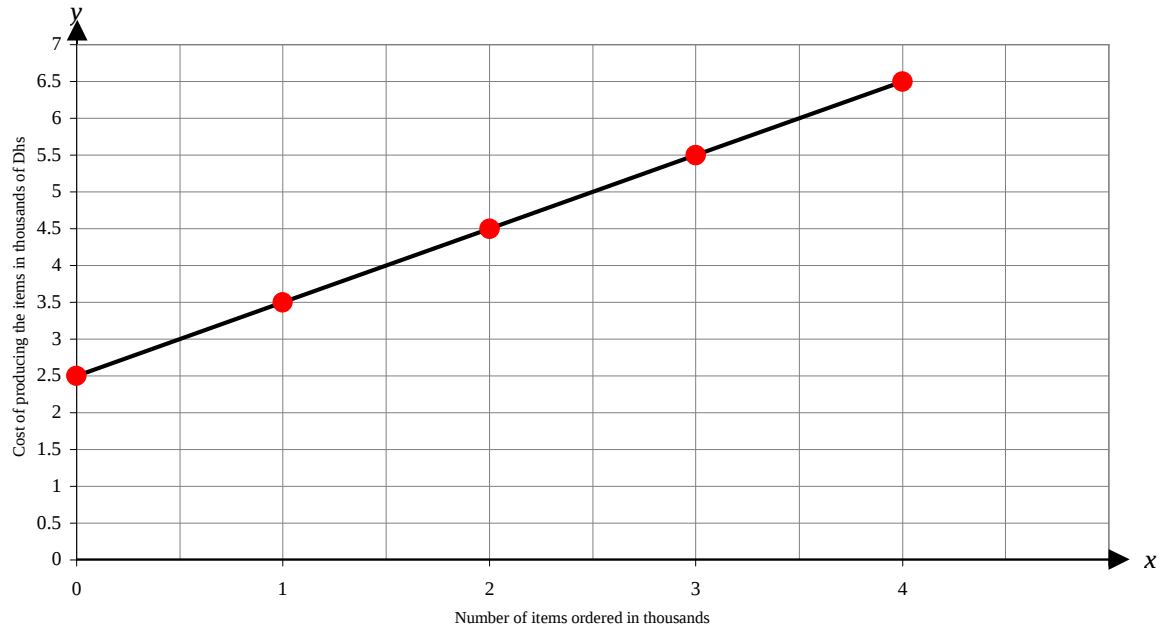


Step 3 The line cuts the y-axis at the point (0, 30). The y-intercept is 30.

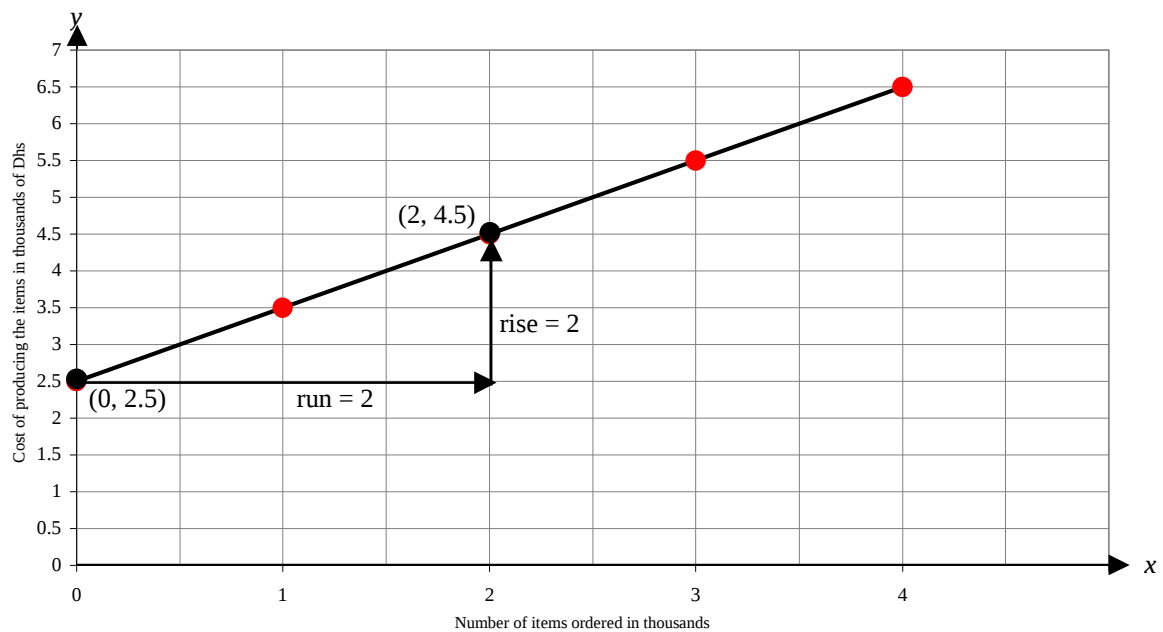
Step 4 $m = -5$ and $b = 30$.
Therefore, the equation of the line is $y = -5x + 30$

Example 9

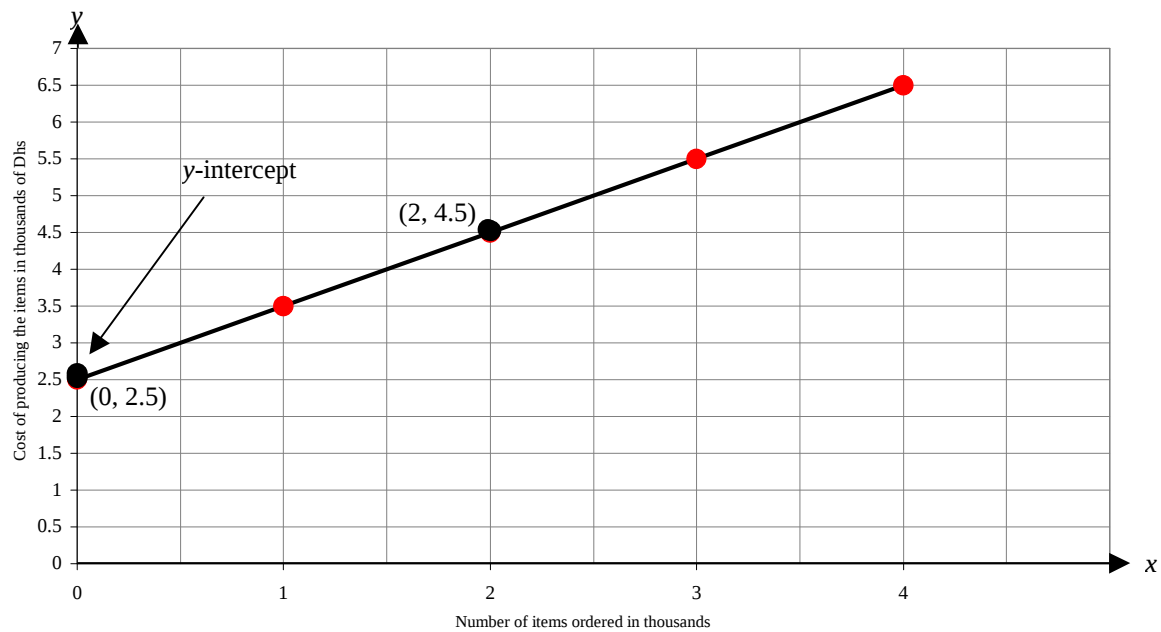
The following graph shows the relationship between the size of an order and the cost of producing the order. Find the slope and the y -intercept. Write the equation of the line.



Step 1 Choose any two points on the line, say $(0, 2.5)$ and $(2, 4.5)$.



Step 2 $y_2 - y_1 = 4.5 - 2.5 = 2$ and $x_2 - x_1 = 2 - 0 = 2$
 The slope $m = \frac{2}{2} = 1$



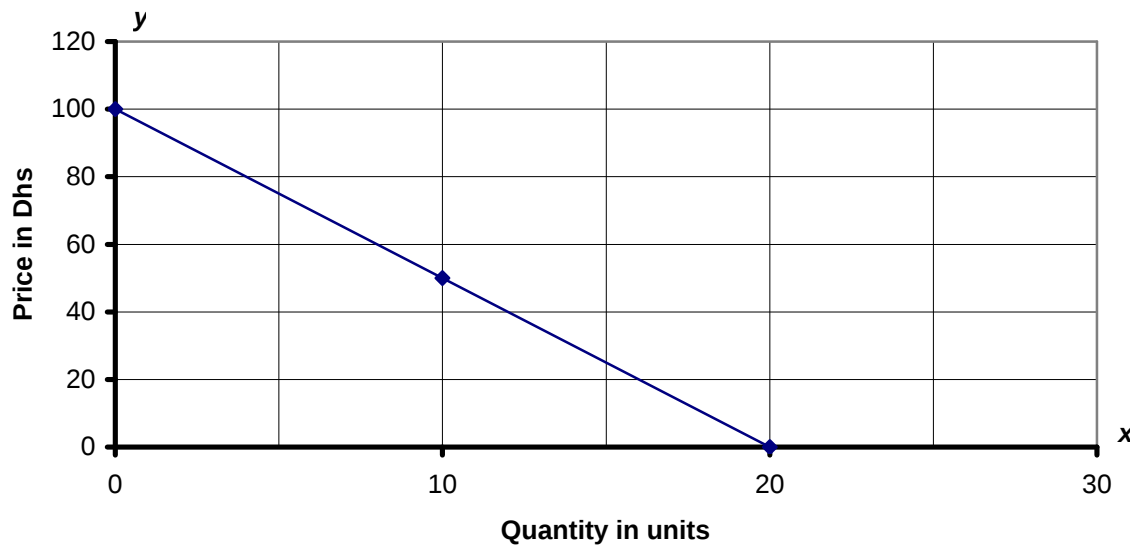
Step 3 The line cuts the y-axis at the point (0, 2.5). The y-intercept is 2.5.

Step 4 $m = 1$ and $b = 2.5$.
 Therefore, the equation of the line is $y = x + 2.5$

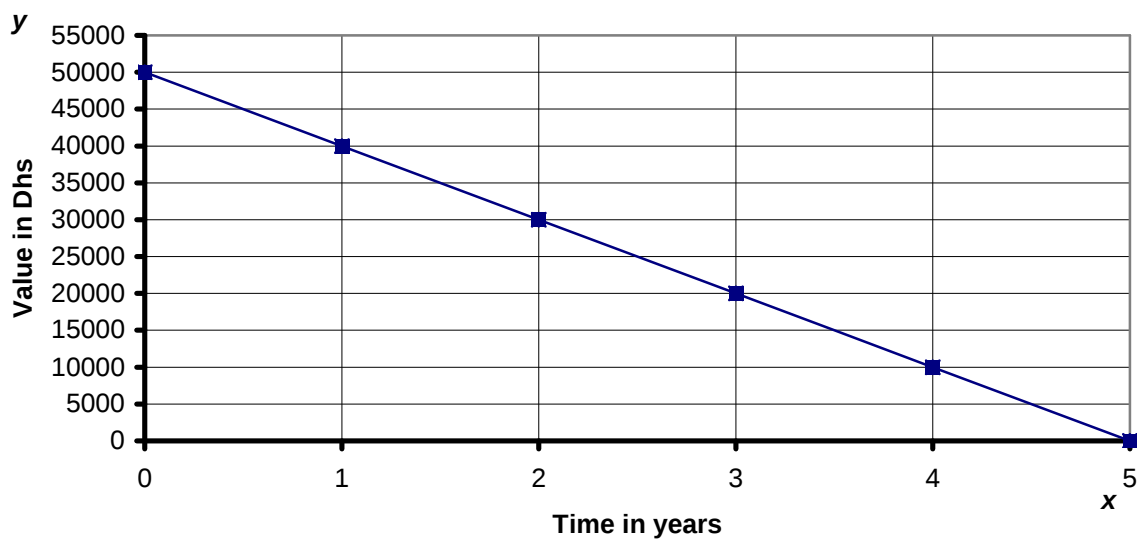
Try these 2

Find the slopes and the y-intercepts of the lines shown below. Write the equations of the lines.

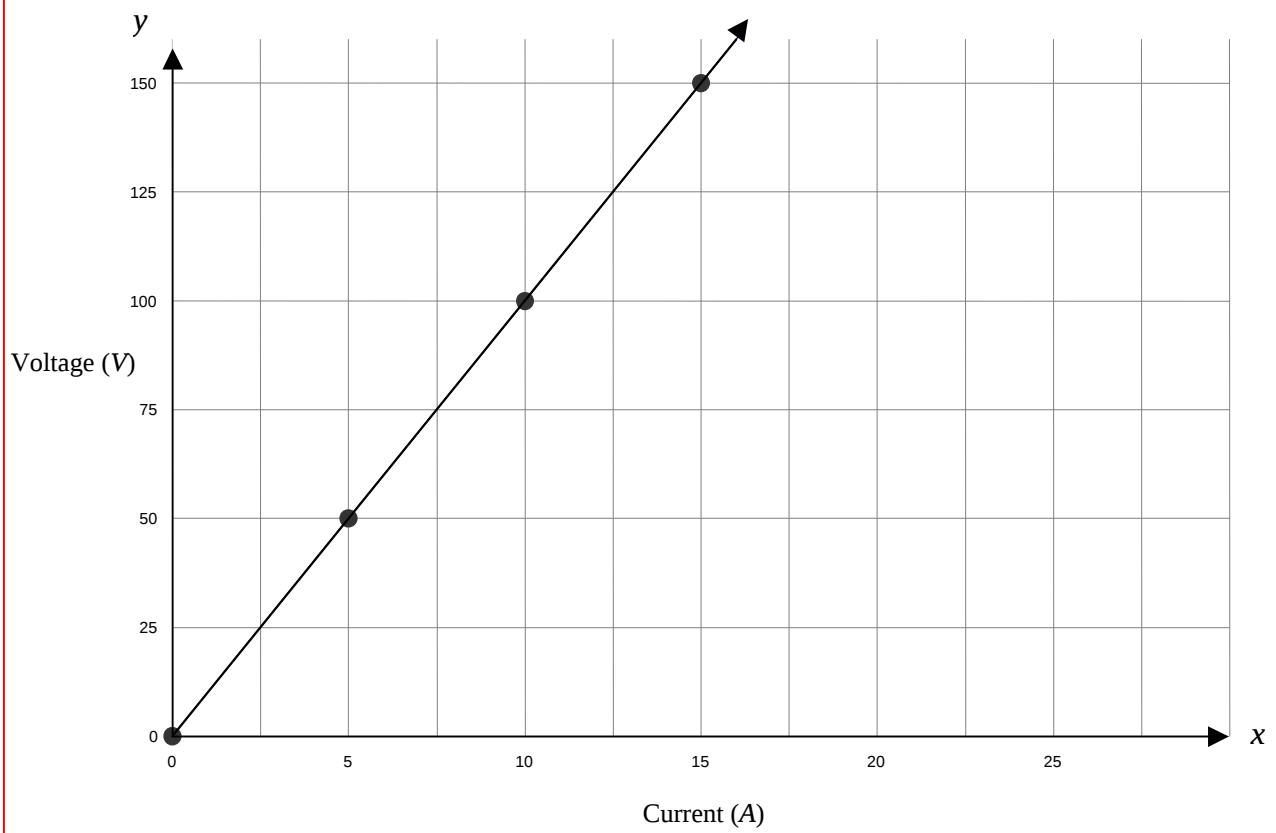
(a)



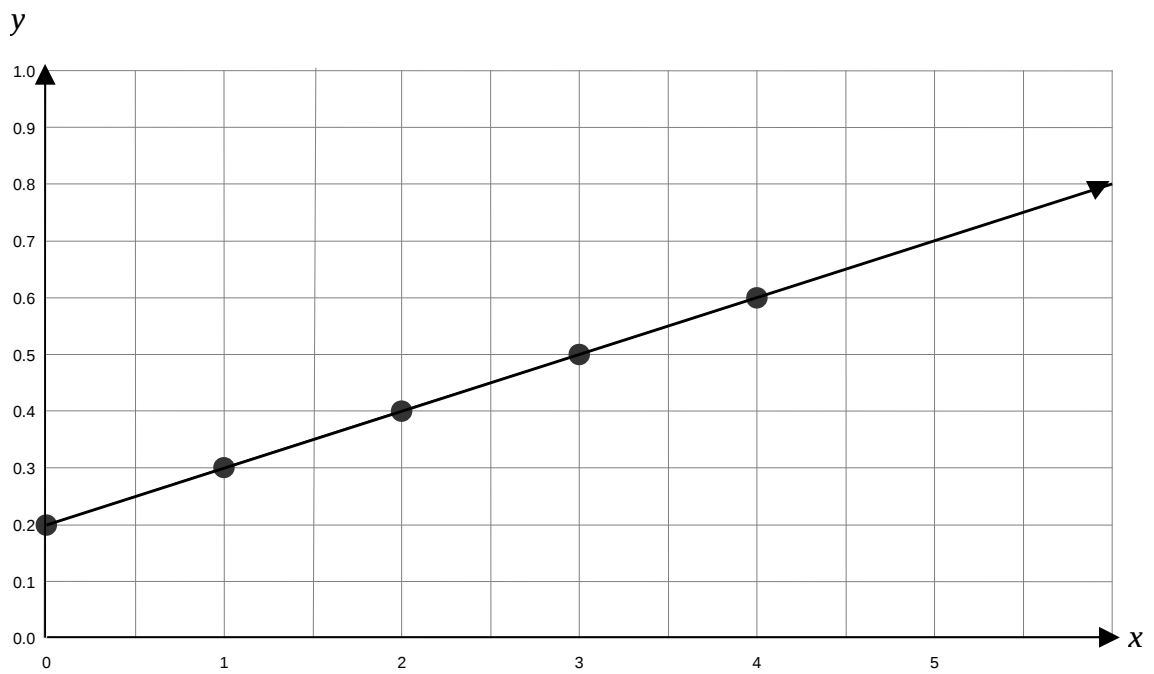
(b)



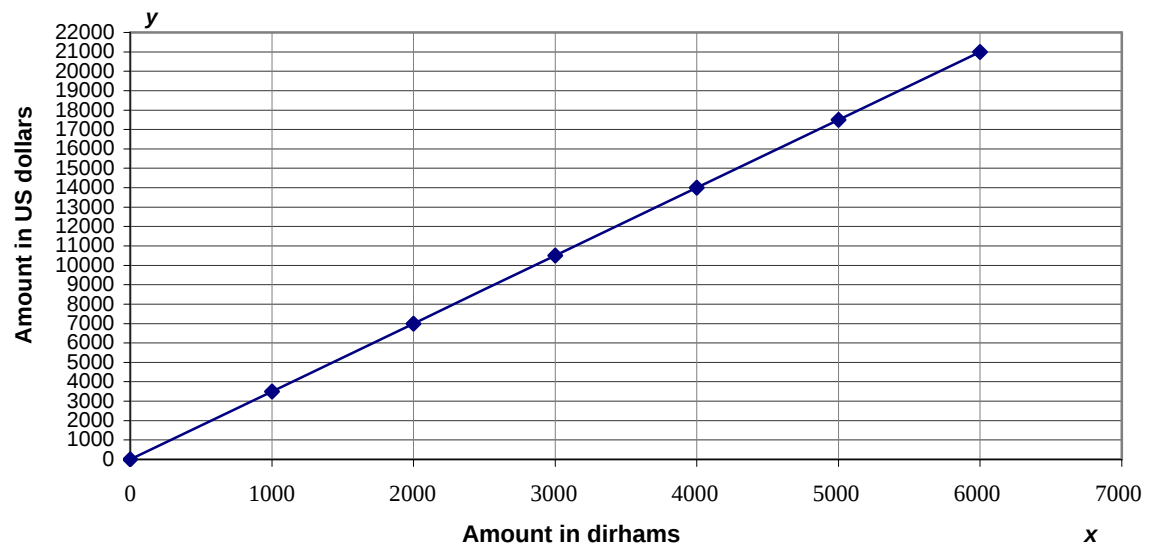
(c)



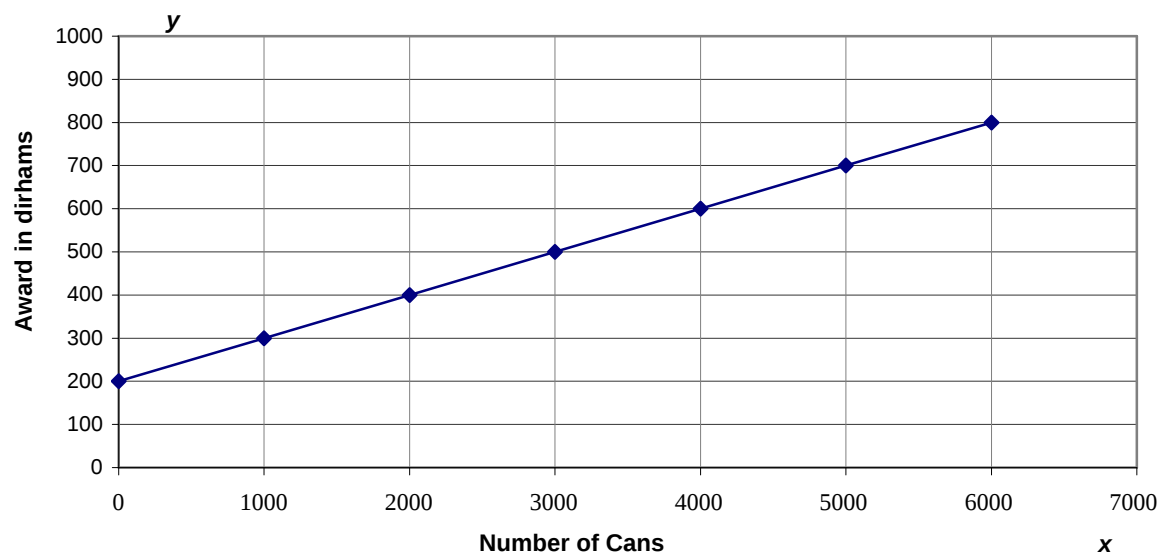
(d)



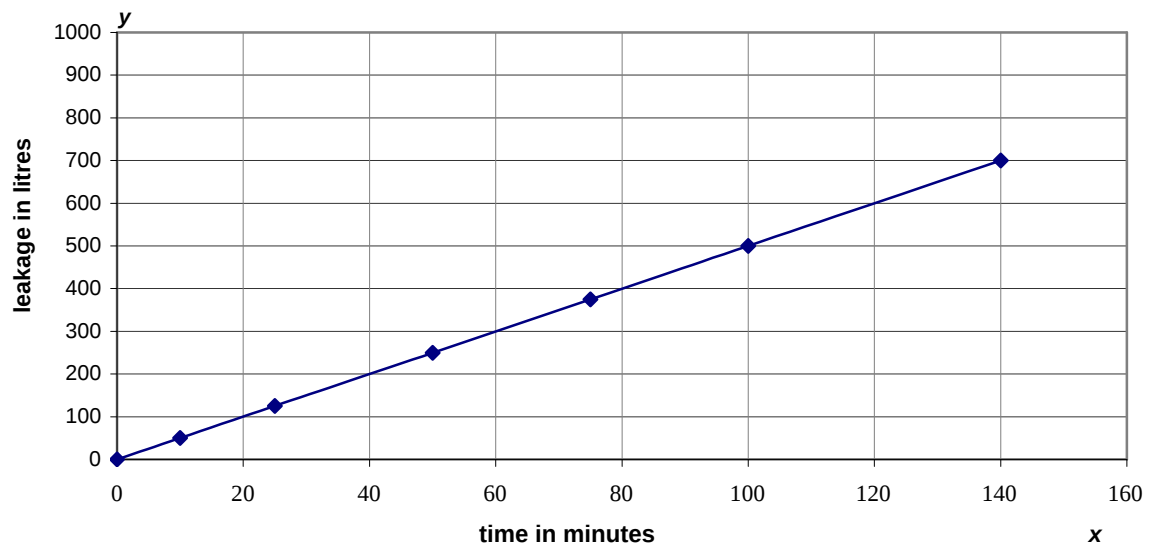
(e)



(f)



(g)



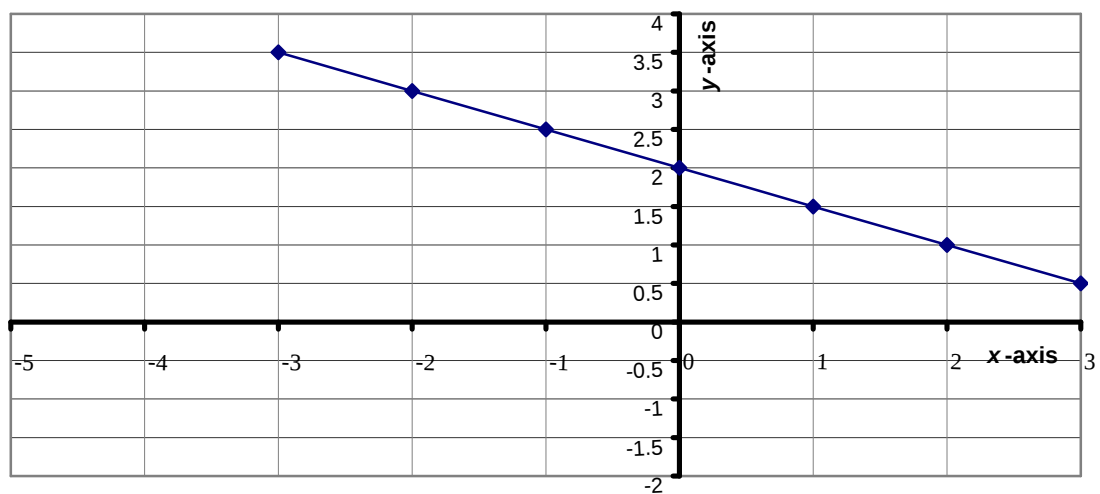
Try these - Answers

- | | | | | |
|---|-----|------------------------|-----|---------------------------|
| 1 | (a) | $y = \frac{5}{3}x - 5$ | (b) | $y = x + 2$ |
| 2 | (a) | $y = -5x + 100$ | (b) | $y = -10\,000x + 50\,000$ |
| | (c) | $y = 10x$ | (d) | $y = 0.1x + 0.2$ |
| | (e) | $y = 3.5x$ | (f) | $y = 0.1x + 200$ |
| | (g) | $y = 5x$ | | |

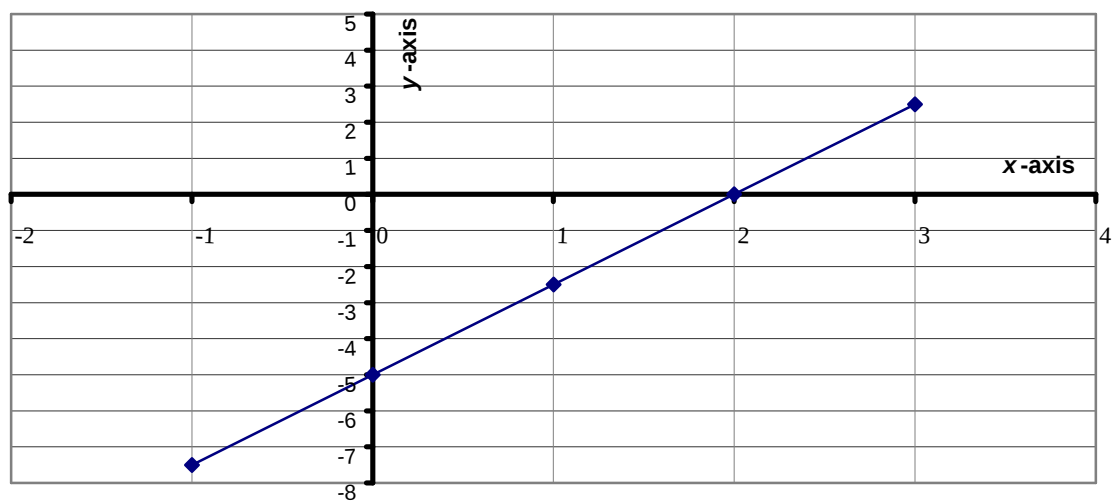
Exercises 9.3

Find the slopes and the y-intercepts of the lines shown below. Write the equations of the lines.

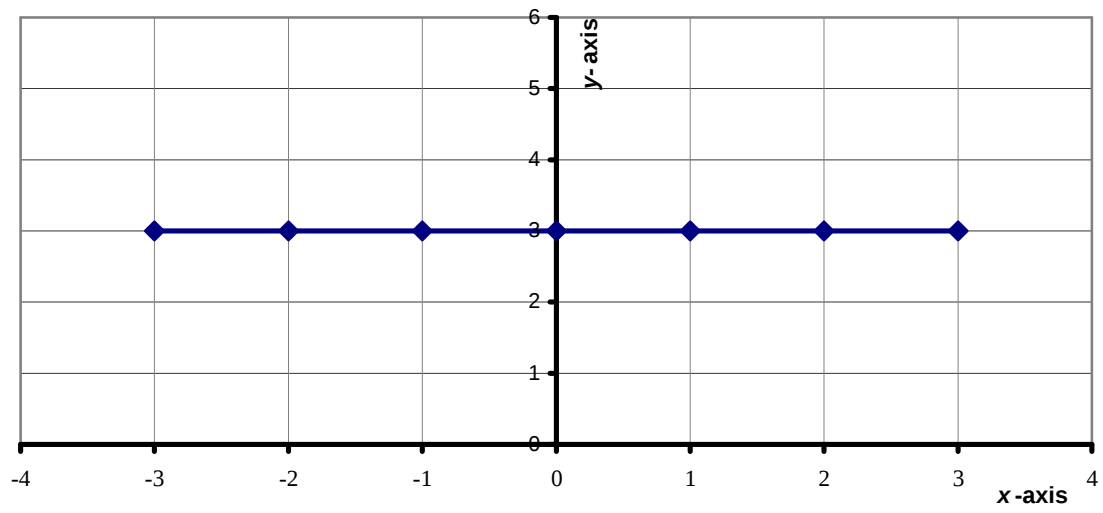
(a)



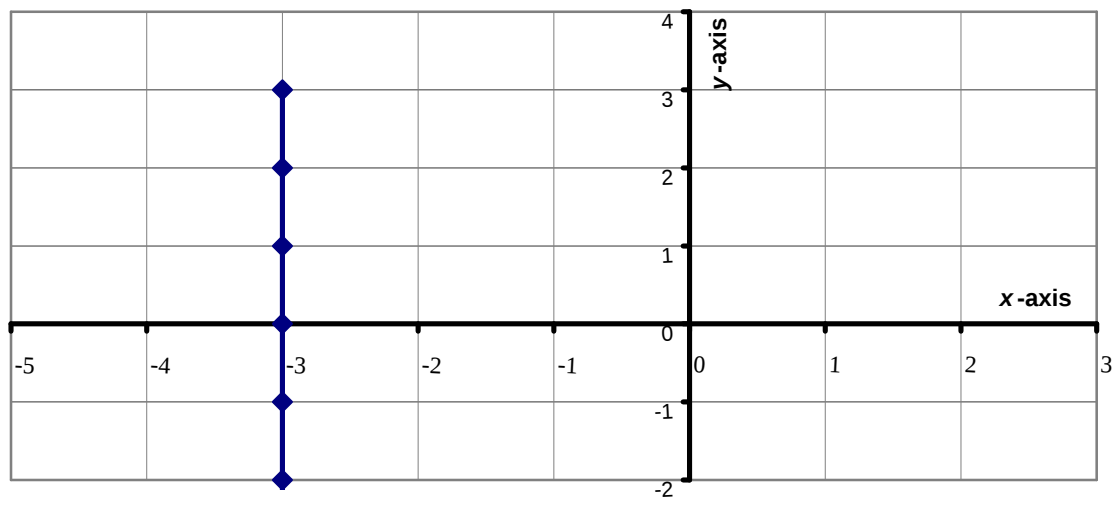
(b)



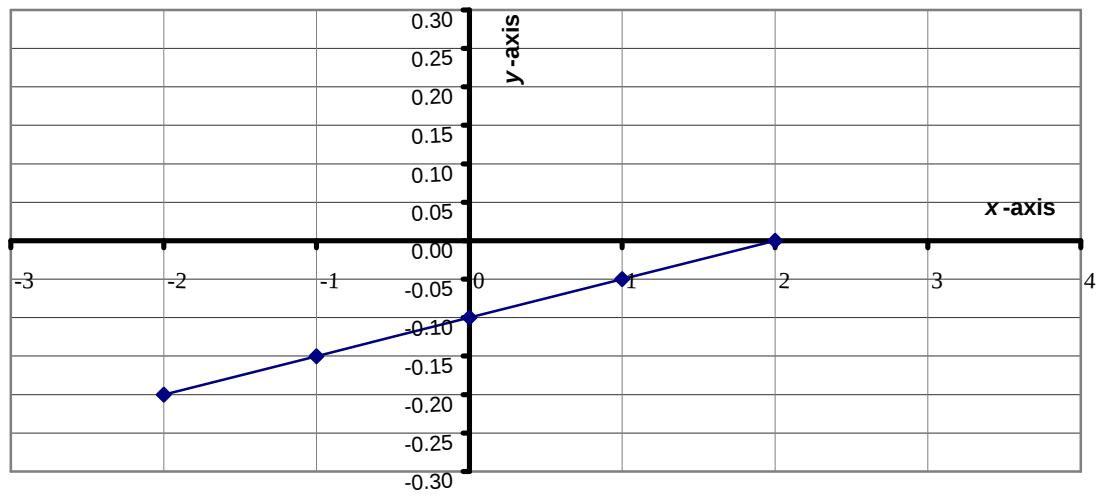
(c)



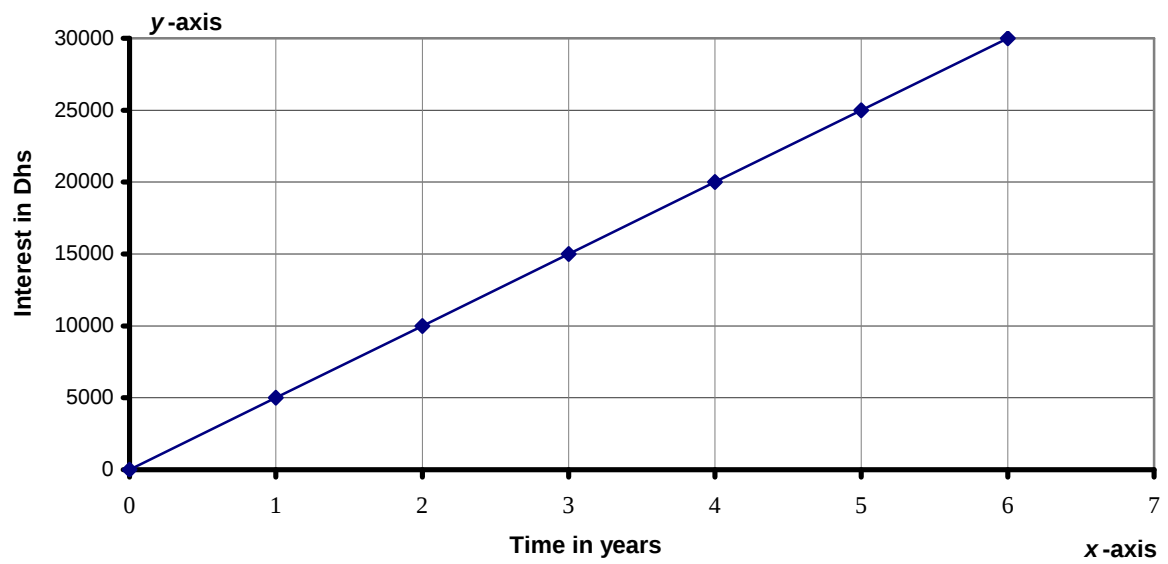
(d)



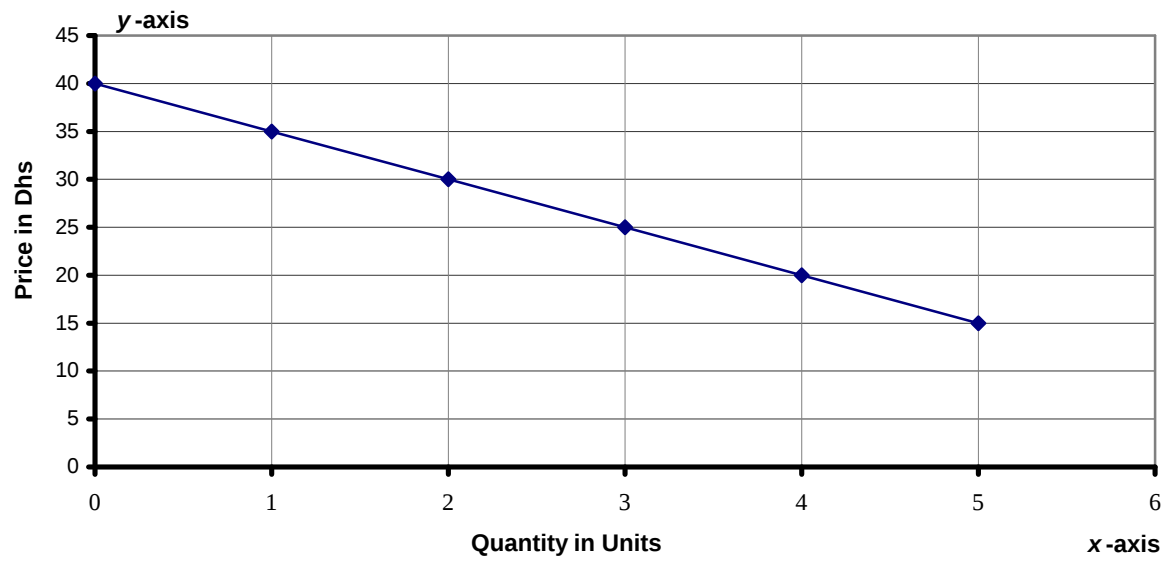
(e)



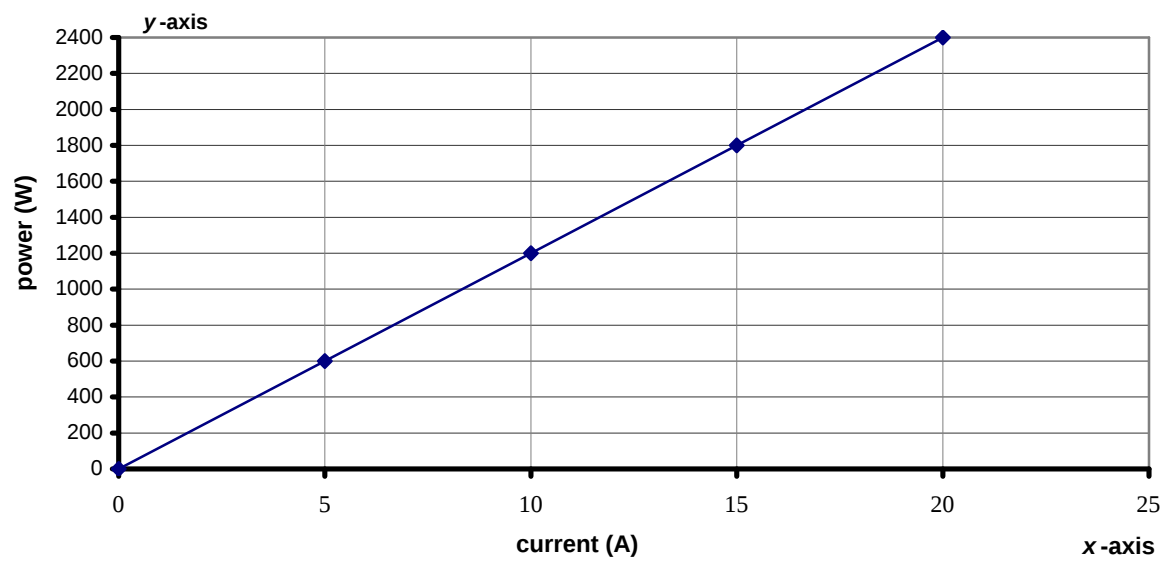
(f)



(g)



(h)



(i)

