مجلس أبو ظبي للتعليم — للصف الحادي عشر العلمي

مدرسة المتنبي للتعليم الثانوي — calculas test — الرياضيات

**السؤال الاول:**

1) Using the first principles method, find the derivative of the function: $f(x) = 3x^2+4$ at $x=2$

$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$f(2) = 3(2)^2 + 4 = 16$

$f(2+h) = 3(2+h)^2 + 4$

$= 3(4+4h+h^2)+4$

$= 16 + 12h + 3h^2$

$= \lim_{h \to 0} \frac{16 + 12h + 3h^2 - 16}{h}$

6

$= \lim_{h \to 0} \frac{3h(4+h)}{h} = 12$

2) An object is dropped from aheight of 64 feet. The height of falling t seconds after release, is given by $g(t) = 64 - 16t^2$ feet.

2

a) Find the average rate of change between $t=1.5$, $t=2$

avf rate $= \frac{g(2) - g(1.5)}{2 - 1.5} = \frac{(64 - 16(2)^2) - (64 - 16(1.5)^2)}{0.5} = \frac{0 - (64-36)}{1/2} = \frac{28}{1/2}$

$= 56$

b) Calculate the gradient of the tangent $g'(t)$ at $t=3$ 2

$g'(t) = -32t \Rightarrow g'(3) = -32(3) = -96$

c) Find the equation of the tangent at $t=3$ 2

at $t=3 \Rightarrow g(3) = 64 - 16(3)^2 = 64 - 144 = -80 \rightarrow (3, -80)$

معادلة المماس $\Rightarrow y - y_1 = m(x - x_1) \Rightarrow (y + 80) = -96(x-3)$

equation of tangent $\Rightarrow y = -96x + 288 - 80 \Rightarrow y = -96x + 208$

3) Applying the chain rule, find $\frac{dy}{dx}$ if $y = 2(5-3x^4)^{-2}$

3

let $u = 5 - 3x^4$

$\frac{du}{dx} = -12x^3$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\Rightarrow y = 2u^{-2}$ $= -4u^{-3} \cdot (-12x^3)$

$\frac{dy}{du} = -4u^{-3}$ $= 48(5-3x^4)^{-3} \cdot x^3 = 48x^3(5-3x^4)^{-3}$

او باستخدام قاعدة القوى

$\frac{dy}{dx} = 2(-2)(5-3x^4)^{-3}(-12x^3)$

$= 48x^3(5-3x^4)^{-3}$

مجلس أبو ظبي للتعليم | | الصف الحادي عشر العلمي
مدرسة المتنبي للتعليم الثانوي | calculas test | الرياضيات

**السؤال الثاني:**

**اولا:**

Differentiate each of these expressions with respect to agiven variable

1) $f(x)=\frac{x^4}{2}-\frac{3}{2}x^2-x$ $\Rightarrow f'(x)=\frac{1}{2}(4x^3)-\frac{3}{2}(2x)-1=2x^3-3x-1$ 3

2) $\frac{d}{dt}\left(3t^{\pi}-\frac{2}{t}+\sqrt{4-t}\right)$ $=3\pi t^{\pi-1}+\frac{2}{t^2}-\frac{1}{2\sqrt{4-t}}$ 3

3) $\frac{dy}{dz}$ , $y=\frac{4z^2-z+3}{\sqrt{z}}$ $\Rightarrow y=z^{-1/2}(4z^2-z+3)=4z^{3/2}-z^{1/2}+3z^{-1/2}$ 3

$\Rightarrow \frac{dy}{dz}=4(\frac{3}{2})z^{1/2}-\frac{1}{2}z^{-1/2}+3(-1/2)z^{-3/2}$

$=6z^{1/2}-\frac{1}{2}z^{-1/2}-3/2\,z^{-3/2}$

4) $\left((x+2)(\frac{x^2-1}{x^2+x})\right)'$ 3

$\left(\cancel{(x+2)}\left(\frac{(x-1)\cancel{(x+1)}}{x\cancel{(x+1)}}\right)\right)=\left(\frac{(x+2)(x-1)}{x}\right)'=\left(\frac{x^2-4}{x}\right)'=(x-4x^{-1})'$

$=(1+4x^{-2})$

**ثانيا:**

Assume that **f & g** are differentiable with

$f(0)=1, f'(0)=-1, f(1)=2, f'(1)=3$

$g(1)=2, g'(1)=-2, g(0)=1, g'(0)=-3$

a) if $h(x)=x^2f(x)$ Find $h'(1)$ 1.5

$h'(x)=2x.f(x)+x^2.f'(x)\Rightarrow h'(1)=2(1).f(1)+1.f'(1)$

$\Rightarrow 2(2)+3=7$

b) If $h(x)=\frac{3}{g(x)}$ Find $h'(0)$ 1.5

$h'(x)=\frac{-3.g'(x)}{(g(x))^2}\Rightarrow h'(0)=\frac{-3.g'(0)}{(g(0))^2}=\frac{-3(-3)}{(1)^2}=9$

مجلس أبو ظبي للتعليم — للصف الحادي عشر العلمي

مدرسة المتنبي للتعليم الثانوي — calculas test — الرياضيـــــات

السؤال الثالث:

Find an antiderivative of each function:

1) $\int\left(\frac{x+e}{2x+2e}\right)dx$

$\int \frac{\cancel{(x+e)}^{1}}{2\cancel{(x+e)}}dx = \int \frac{1}{2}dx = \frac{1}{2}x + C$ 3

2) $\int\left(\frac{x^{-3}}{3}+x\right)dx$ 3

$\int \frac{1}{3}x^{-3}dx + \int x\,dx$

$\frac{1}{3}\cdot\frac{x^{-2}}{-2} + \frac{1}{2}x^2 + C = -\frac{1}{6}x^{-2} + \frac{1}{2}x^2 + C$

3) $\int 2x\left(4-x^2+\sqrt{x}\right)dx$ 3

$\int\left(8x - 2x^3 + 2x^{3/2}\right)dx = \frac{8x^2}{2} - \frac{2x^4}{4} + 2\frac{x^{5/2}}{5/2} + C$

$= 4x^2 - \frac{1}{2}x^4 + \frac{4}{5}x^{5/2} + C$

4) $\int\frac{(2x^2-1)^2}{2x^2}dx$ 3

$\int \frac{4x^4-4x^2+1}{2x^2}dx = \frac{1}{2}\int x^{-2}(4x^4-4x^2+1)dx$

$\Rightarrow \frac{1}{2}\int\left(4x^2-4+x^{-2}\right)dx$

$\Rightarrow \frac{1}{2}\left(\frac{4x^3}{3} - 4x + \frac{x^{-1}}{-1}\right) + C$

5) $3\int\frac{d}{dx}\left(1-\sqrt{x}\right)dx$ 3

$\Rightarrow \frac{2}{3}x^3 - 2x - \frac{1}{2}x^{-1} + C$

$3(1-\sqrt{x}) + C$